PROGRESSIVE SCHOOL SERIES.

COMMON SCHOOL ARITHMETIC

(In Three Parts).

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Prescribed for use in the Public Schools in Nova Scotia and New Brunswick.

"Long is the road by rules, short and easy by examples."

SENECA.

PART TWO.

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CONTENTS.

FACTORS AND MULTIPLES.

1. ORAL.

1. What is the product of 3 and 4? Of 2, 2 and 3?

2. 12 is the product of what two numbers? Of what three?

3. 16 is the product of what two numbers? Of what other two? Of what three? Of what four?

4. Name the factors of 6. Of 15.

5. Resolve 8 into two factors. Now into three.

6. $1 \times 7 = ?$ Can you name two other numbers whose product is 7?

7. Name some other number that has no factors except itself and 1.

2. Numbers are **Whole, Fractional** or **Mixed.**
6, 90, 100 are **Whole Numbers.**
$\frac{3}{4}$, $\frac{5}{8}$, $\frac{19}{100}$ are **Fractional Numbers.**
$6\frac{3}{8}$, $5\frac{3}{17}$, $1\frac{11}{19}$ are **Mixed Numbers.**

3. Whole numbers are sometimes called integral numbers, and any whole number is called an integer. 16, 18, 24 are integers.

4. 1 is often called unity.

5. An **Even Number** is one that can be divided into two equal whole numbers.

6. An **Odd Number** is one that cannot be divided into two equal whole numbers.

7. When a number is the product of two or more numbers each of the latter is a **Factor** of that number. Since $30 = 2 \times 3 \times 5$, 2, 3 and 5 are factors of it.

8. A **Composite Number** is one that has other factors besides itself and unity. 6, 12, 20, are composite numbers. Why?

9. A **Prime Number** has no factors besides itself and unity. 2, 3, 5, 7 are prime numbers. Why?

Two numbers are said to be prime to each other when they have no common factor (except unity). 16 and 9 are prime to *each other* (although they are both composite numbers) because they have no factor common to both.

10. The **Prime Factors** of a number are the prime numbers whose continued product is that number. 3 and 6, or 9 and 2 are factors of 18, but its prime factors are 2, 3 and 3.

Write the twelve prime numbers from

1. 1 to 32. 2. 32 to 84. 3. 84 to 150.

11. Find the prime factors of 30.

EXAMPLE 1.

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

2 is a prime factor of 30; 3 is a prime factor of 15 and 30, and 5 is a prime factor of 5, 15 and 30.

Divide the given number by any prime number that will exactly divide it. Divide the quotient by a prime number, and so continue until the quotient is 1. The several divisors will be the prime factors.

In practice, it is better to begin with the smallest prime factor that can be used. 2 is a prime factor of all even numbers.

EXERCISE I.

What are the prime factors of:

1.	8.	4.	18.	7.	26.	10.	39.	13.	114.	16.	450.
2.	10.	5.	21.	8.	27.	11.	48.	14.	243.	17.	420.
3.	12.	6.	24.	9.	33.	12.	153.	15.	200.	18.	690.

12. Find the prime factors of 5005.

EXAMPLE 2.

$$\begin{array}{r|l} 5 & 5005 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

Here the smallest prime factor of 5005 is 5; the smallest prime factor of 1001 is 7; and the smallest of 143 is 11, leaving 13 for the remaining prime factor.

Therefore the prime factors are 5, 7, 11 and 13. It is evident that the same result would be attained by dividing first by any of the prime factors and continuing in any order.

EXERCISE II.

Find the prime factors of:

1.	300.	7.	1025.	13.	1067.	19.	7128.	25.	27573.
2.	330.	8.	1024.	14.	1183.	20.	7854.	26.	7259.
3.	390.	9.	1033.	15.	1625.	21.	5987.	27.	70191.
4.	510.	10.	1140.	16.	2051.	22.	5625.	28.	18746.
5.	999.	11.	1096.	17.	3809.	23.	9257.	29.	10403.
6.	1001.	12.	1157.	18.	6381.	24.	16128.	30.	66953.

13. ORAL.

1. What number will exactly divide 6? What other?
2. What is the product of the two divisors of 6?
3. What other name than divisors can you give to the divisors of 6?
4. What number will exactly divide 6 and 8?
5. Name a common divisor of 6 and 9.
6. Name a common divisor of 8 and 12. Name another.
7. What is the greatest common divisor of 8 and 12?
8. Name a common divisor of 10 and 15.
9. Name a common divisor of 20 and 30. Name another common divisor of 20 and 30. What is the greatest common divisor of 20 and 30?
10. What is the greatest common divisor of 8, 12 and 16? Of 12, 18 and 24? Of 14, 21, 28? Of 50, 75 and 100?
11. Every exact divisor of a number is a —— of that number?

14. A **Divisor** of a number is one of the whole numbers which, being multiplied together, will produce that number.

Every divisor of a number is a factor of it. Divisor must be here understood to mean exact divisor.

15. A **Common Divisor** of two or more numbers is a number that is a divisor of each of them.

16. The **Greatest Common Divisor** of two or more numbers is the greatest number that is a divisor of each of them.

Greatest Common Divisor is sometimes called Highest Common Factor, and sometimes Greatest Common Measure.

17. To find the G. C. D. of small numbers.

What is the greatest common divisor of 24, 60 and 84?

EXAMPLE 3.

$$\begin{array}{r|l} 2 & 24,\ 60,\ 84 \\ \hline 2 & 12,\ 30,\ 42 \\ \hline 3 & 6,\ 15,\ 21 \\ \hline & 2\quad 5\quad 7 \end{array}$$

2 is a common divisor of 24, 60 and 84; the second 2 is a common divisor of the quotients 12, 30 and 42, and consequently of 24, 60 and 84; 3 is a common divisor of 6, 15 and 21, and consequently of the numbers in the second and first lines. Therefore $2 \times 2 \times 3 =$ G. C. D.

G. C. D. $= 2 \times 2 \times 3 = 12$.

18. Hence, to find the G. C. D. of two or more numbers, we write the numbers in a line with a vertical line to the left, and divide by any divisor common to all the numbers. Divide the quotients in the same manner and

continue the process until the quotients have no common divisor. The continued product of the divisors will be the G. C. D.

EXERCISE III.

Find the G. C. D. of:

1. 8 and 12.
2. 9 " 12.
3. 10 " 15.
4. 12 " 18.
5. 16 " 24.
6. 21 " 28.
7. 27 " 36.
8. 22 " 33.
9. 26 " 39.
10. 55 " 77.
11. 24, 36, 48.
12. 36, 40, 72.
13. 33, 231, 66.
14. 84, 126, 210.
15. 44, 110, 77.
16. 75, 250, 400.
17. 144, 576, 720.
18. 176, 440, 1100.
19. 81, 990, 1080.
20. 390, 672.
21. 728, 455.
22. 36, 48, 60, 72, 540.
23. 400, 880, 1320.
24. 330, 420, 1650.
25. 390, 468, 780.
26. 121, 1100, 9999, 1001.
27. 686, 1029, 2401.
28. 1680, 5050, 5390.
29. 112, 147, 168, 189.
30. 576, 672, 864, 1132.

To find the G. C. D. of large numbers.

19. If we examine any two numbers, say 18 and 48, we shall find that their G. C. D. is also the G. C. D. of the smaller number and the difference between the larger and any multiple of the smaller.

6 is the G. C. D. of 18 and 48. It is also the G. C. D. of 18 and 12, 12 being the difference between 48 and 36, 36 being a multiple of 18.

Find the G. C. D. of 405 and 900.

EXAMPLE 4.

405 | 900 | 2
810

90 | 405 | 4
360

45 | 90 | 2
90

The G. C. D. of 405 and 900 is the G. C. D. of 405 and 90. The G. C. D. of 90 and 405 is the G. C. D. of 90 and 45. The G. C. D. of 45 and 90 is 45. Therefore 45 is the G. C. D. of 405 and 900.

20. To find the G. C. D. of two numbers, divide the greater by the less, then the less by the remainder, then the last divisor by the last remainder, and continue the process until nothing remains. The last divisor is the G. C. D.

21. When there are more than two numbers find the G. C. D. of two of the numbers, then of the common divisor thus found and a third number, and so on with a fourth, etc. The last divisor found will be the G. C. D. of all the numbers.

EXERCISE IV.

Find the G. C. D. of:

1. 38 and 133.
2. 115 " 161.
3. 154 " 253.
4. 333 " 592.
5. 336 " 812.
6. 407 " 1067.
7. 679 " 1869.
8. 331 " 509.
9. 810 " 2700.
10. 341 " 1089.
11. 1226 " 2013.
12. 825 " 1372.
13. 2041 " 8476.
14. 2373 " 6667.
15. 231 " 609.
16. 585 and 1729.
17. 6006 " 3318.
18. 2021 " 6493.
19. 8383 " 9589.
20. 141, 799, 940.
21. 1435, 2168, 2135.
22. 575, 805, 4530.
23. 1265, 1771, 35420.
24. 8604, 17064, 11111.
25. 25749, 37899.
26. 927, 1233, 14786.
27. 6381468, 7470036
28. 100064, 3880612.
29. 378107, 578708.
30. 36420, 3338500.

31. A farmer has two logs, one 30 feet long and the other 35 feet long. He wishes to saw them into lengths to make pickets for a fence. What is the greatest length he can make the pickets so as to use both logs and not waste any of the timber?

32. A grocer has 54 lbs. of black tea and 96 lbs. of green. He wants to do up each kind in packages of the same weight. What is the largest package he can make so as to have none left over?

33 What is the greatest length of flooring that can be used, without cutting, for three rooms that are, respectively, 24, 36 and 48 feet long?

34. A boy has 190 yards of red cord and 230 yards of white. He wishes to divide both into fishing lines so that a red line will be as long as a white one. What is the greatest length he can make the lines?

35. A field is 900 feet long and 480 feet broad. What is the length of the longest pole that can be used to exactly measure both the length and breadth of it?

36. A, B and C bought cattle at the same price per head. A paid $405 for his lot, B $900 for his, and C $1085 for his. What was the largest price per head they could have paid?

22. ORAL.

1. Name three numbers that 2 will exactly divide.
2. " " " " 3 " " "
3. " " " " 4 " " "
4. Such numbers are called multiples of the numbers that exactly divide them.
5. Name the numbers under 18 that are multiples of 2.
6. " " " " 24 " " " " 3.
7. " " " " 32 " " " " 5.
8. Name a number that is a multiple of both 2 and 3. Name another. Name a common multiple of 3 and 4. Name another. What is the least common multiple of 3 and 4. What is the least common multiple of 3 and 5.
9. Name a common multiple of 3, 5 and 6.
10. Name all the numbers under 160 that are common multiples of 3, 5 and 6. What is the least common multiple of 3, 5 and 6.
11. Name two multiples of 6. Name two more.
12. How many multiples can a number have?

23. A **Multiple** of a number is a number that is exactly divisible by that number. 18 is a multiple of 3. Why?

A *Multiple* of any number is obtained by *multiplying* that number by any number greater than 1.

24. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of them. 20 is a common multiple of 4 and 5. Why?

25. The **Least Common Multiple** of two or more numbers is the smallest number that is exactly divisible by

each of them. 30 is the least common multiple of 5, 6 and 10. Why?

26. Find the L. C. M. of 18, 30 and 56.

EXAMPLE 5.

$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$
$56 = 2 \times 2 \times 2 \times 7$

L. C. M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

The L. C M. of all the numbers must be a multiple of any factor of any one of them, therefore the L. C. M. must contain the factors 2, 3, 5 and 7.

2 must appear three times as a factor of the L. C. M. since it occurs three times as a factor of 56; 3 must appear twice as a factor of the L. C. M. since it occurs twice as a factor of 18; 5 must appear once as a factor of the L. C. M. since it occurs once; 7 must appear once as a factor of the L. C. M. as it occurs only once.

27. Hence, to find the L. C. M. of two or more numbers, we find the prime factors of the numbers, and take the product of these factors, using each the greatest number of times it appears as a factor of any of the numbers.

28. The following method is much used:

EXAMPLE 6.

2	18, 30, 56
2	9, 15, 28
3	3, 5, 28

L. C. M. $= 2 \times 3 \times 3 \times 5 \times 28 = 2520$

Arrange the numbers as in example.

Divide by the smallest prime factor common to any two of the numbers, and place the quotients and any undivided number in the line below. Continue in the same way with the quotients and any undivided number brought down. When the quotients and any undivided number brought down have no divisor common to any two of them, the product of the divisors, the final quotients, and any undivided number brought down, will be the L. C. M.

EXERCISE V.

Find the L. C. M. of:

1. 16, 24, 32.
2. 24, 36, 48.
3. 30, 45, 180.
4. 14, 21, 28.
5. 18, 24, 36, 48.
6. 14, 18, 20, 21.
7. 45, 54, 63.
8. 3, 4, 5, 6, 7.
9. 2, 3, 5, 7.
10. 26, 39, 52.
11. 22, 33, 44.
12. 34, 51, 68.
13. 45, 60, 50, 63, 84.
14. 121, 132, 144.
15. 22, 88, 132, 198.
16. 21, 22, 24, 26, 28, 30.
17. 98, 154, 198, 284.
18. 72, 108, 180, 252.

19. What is the smallest sum of money with which I can buy steers at \$16 each, or oxen at \$36 each, or Jersey cows at \$45 each, and spend all my money each time?

20. What is the shortest cord that can be cut up into lines 20 feet, 30 feet, or 40 feet long?

21. A farmer has barrels that hold 3 bushels each and boxes that hold 4 each; what is the smallest quantity of oats that can be exactly measured with either a barrel or a box?

22. What is the smallest number that can be exactly divided by 12, 18 or 24 and have a remainder of 5 each time?

23. What is the product of the sum and difference of the H. C. F. and L. C. M. of 10, 25 and 45?

EXERCISE VI.

Miscellaneous Examples:

1. What is the G. C. D. of 24, 36 and 28? The L. C. M.?

2. What is the G. C. D. of 26, 30, 39, 60?

3. L. C. M. of 1, 2, 3, 4, 5, 6, 7, 8, 9?

4. L. C. M. of 22, 28, 40, 52?

5. G. C. D. of 680, 850, 2550?

6. G. C. D. of 618, 7210, 9270?

7. G. C. D. of 4341, 6117?

8. L. C. M. of 850, 2300, 2900?

9. L. C. M. of 44, 126, 198, 280?

10. L. C. M. of 50, 169, 675, 702?

11. G. C. D. of 232, 696, 3480, 24360?

12. G. C. D. of 5950, 41650, 65450?

13. G. C. D. of 9365706, 111111111?

14. G. C. D. of 236785644, 375686064?

15. Divide the L. C. M. of 34, 102 and 323 by their G. C. D.

EXAMINATION PAPER. No. 1.

(TIME: ONE HOUR. VALUE OF EACH QUESTION: 10).

1. Bought 3 yards of lace at 45 cents a yard, 4 pairs of socks at 25 cents a pair, 6 handkerchiefs at 35 cents each, and 1 doz. of collars at 15 cents each. I gave a ten dollar bill in payment. How much change was coming to me?

2. If 17 yards of muslin cost $5.10, what must be paid for 37 yards?

3. Paid 5 cents a pound for dates and sold them at 10 cents a pound and gained $2. How many pounds did I sell?

4. If I buy 100 lbs. of raisins at 8 cents a pound what must I get a pound for them to gain $3 on the lot?

5. A grocer mixed 60 lbs. of tea at 20 cents a pound with 40 worth 30 cents a pound. At how much per pound must he sell the lot to gain $5?

6. The hay on a farm was worth $500, the stock was worth twice as much as the hay, and the farm was worth three times as much as the stock. What were they all worth?

7. If 5 pigs be worth 4 sheep, and 3 sheep be worth $7.50, what is a pig worth?

8. In a school there were as many boys as girls. The teacher divided 120 apples among them, giving each boy 2 and each girl 3. How many pupils were there?

9. Write any three prime numbers between 354 and 430.

10. Find the greatest number that will divide 10974 and 15336, leaving remainders respectively 54 and 36.

FRACTIONS.

29. ORAL.

1. If I divide an apple into two equal parts, what is each part called?

2. If I divide an apple into three equal parts, what is each called?

3. If I divide anything into four equal parts, what is each called?

4. What is the half of 12 cents? What is one-third of 12 cents? How much is two-thirds of 12 cents?

5. What is one-fourth of 16 cents? How many cents in three-quarters of 16 cents?

6. A pie was divided equally among 5 boys. What part of it did each get? A cake was divided equally among 6 boys. What fraction of it did each get?

7. How many halves in a whole apple? How many thirds? How many fourths?

8. Into how many equal parts would you divide anything to get sevenths? To get eighths?

9. Which are the greater, halves or fourths? Halves or thirds? Thirds or fourths?

30. When we divide anything—for instance, an orange—into:

2 equal parts, each part is called a half, and is written $\frac{1}{2}$.
3 " " " " " a third, " " $\frac{1}{3}$.
4 " " " " " a fourth, " " $\frac{1}{4}$.
7 " " " " " a seventh, " " $\frac{1}{7}$
10 " " " " " a tenth, " " $\frac{1}{10}$

It is apparent that 2 halves = 1 whole.
" " " 3 thirds = 1 "
" " " 4 fourths = 1 "
" " " 7 sevenths = 1 "
" " " 10 tenths = 1 "

If, when we divide the orange into 3 equal parts, we take 2 of these parts, we take 2 thirds of it, which is written $\frac{2}{3}$.

When we divide it into 4 equal parts, and take 3 of them, we take 3 fourths of it, which is written $\frac{3}{4}$

If we divide anything into, say, 7 equal parts, and take 5 of them, we take 5 sevenths, which is written $\frac{5}{7}$.

Such expressions are called fractions.

? A Fraction is a number denoting one or more equal parts of a unit.

32. ORAL.

1.	How many	thirds	in 1?	2?	3?	6?
2.	"	"	fourths	" 2?	5?	3?	7?
3.	"	"	fifths	" 1?	9?	10?	11?
4.	"	"	sixths	" 2?	7?	8?	20?
5.	"	"	sevenths	" 3?	8?	20?	40?
6.	"	"	fourteenths	" 1?	3?	5?	7?

Write in figures:

1. one sixth.
2. three fifths.
3. four ninths.
4. five tenths.
5. seven elevenths.
6. ten sixteenths.
7. eight twenty-firsts.
8. twenty ninths.
9. ninety two-hundreths.

EXAMPLE 7. If $\frac{3}{4}$ of a pound of tea cost 30 cents, what will 1 pound cost?

$\frac{3}{4}$ cost 30c.

$\frac{1}{4}$ costs $\frac{30c.}{3}$

$\frac{4}{4}$ cost $\frac{30c. \times 4}{3} = 40$ cents.

Since three fourths ($\frac{3}{4}$) of a lb cost 30 cents, one fourth ($\frac{1}{4}$) will cost $\frac{1}{3}$ of 30 cents, which is indicated by the expression $\frac{30c.}{3}$, that is $\frac{30c.}{3}$ or 10 cents is the value of $\frac{1}{4}$. $\frac{4}{4}$ will cost four times as much as $\frac{1}{4}$, that is $\frac{30c. \times 4}{3}$ which is equal to 40 cents. But $\frac{4}{4}$ is equal to 1, therefore 1 will cost 40 cents.

If $\frac{9}{10}$ of a barrel of sugar cost \$18.36, what will a whole barrel come to?

\$18.36 = 1836 cents.

$\frac{9}{10}$ of a barrel cost 1836 cents.

$\frac{1}{10}$ " " costs $\frac{1836c}{9}$

$\frac{10}{10}$ " " cost $\frac{1836c. \times 10}{9} = 2040$ cents = \$20.40.

Many of the following, if not all, can be worked by the pupil without the aid of slate and pencil. The pupil should explain the process of every exercise. Thus in the question, "If $\frac{5}{7}$ of a ton of coal costs \$4.50, what will 1 ton cost?" the pupil should proceed much in this way:

Five sevenths of a ton cost \$4.50 or 450 cents.
One seventh of a ton costs 90 cents.
Seven sevenths of a ton cost 630 cents or \$6.30.

EXERCISE VII.

1. If $\frac{2}{3}$ of a bushel of oats cost 30 cents, what will a whole bushel cost?

2. If $\frac{4}{5}$ of a ton of coal cost \$4.40, what will 1 ton come to?

3. Paid \$3.60 for $\frac{5}{6}$ of a barrel of flour, what is that a barrel?

4. A boy owned a boat and sold $\frac{3}{8}$ of it for **$9.60**; what was the boat worth?

5. $\frac{5}{7}$ of a spar measured sixty feet, how long was the whole spar?

6. $\frac{3}{8}$ of a yd. of tweed cost 60 cents, what will 2 yds. come to? 3? 5?

7. Paid $4.80 for $\frac{5}{16}$ of a ton of hay; how much was that a ton?

8. A man sold $\frac{7}{8}$ of his farm for $1400; what was the value of what he had left?

9. A man after spending $\frac{7}{12}$ of his money had $15 left; how much had he at first?

10. $\frac{7}{11}$ of a piece of work can be done by 14 men in a day; how many men can do the whole of it in the same time?

11. $\frac{11}{12}$ of a barrel of flour will feed 22 men for a week; how many men will a whole barrel feed for the same time?

12. A company of soldiers eat up $\frac{17}{30}$ of a quantity of provisions in 34 days; how long would the whole of the provisions last them?

13. A boy had 80 marbles and gave away $\frac{7}{20}$ of them; how many had he left?

14. A had 80 cents, B $\frac{5}{8}$ as many as A, and C $\frac{4}{5}$ of B's lot; how many had C?

15. Divide a dollar among A, B and C so as to give A $\frac{5}{8}$, B $\frac{2}{8}$ and C the rest.

EXERCISE VIII.

EXAMPLE 8. If a cord of wood cost $4.40, what will $\frac{3}{8}$ of a cord come to?

1 cord $= \frac{8}{8}$ of a cord

$\frac{8}{8}$ of a cord costs 440 cents

$\frac{1}{8}$ " " $\frac{440}{8}$ "

$\frac{3}{8}$ " " $\frac{440 \times 3}{8} = 165$ cents $= \$1.65$.

1. If a yd. of silk cost $3.60, what will $\frac{3}{5}$ of a yd. cost?

2. A ton of coal is worth $5.76, what must I pay for $\frac{5}{6}$ of a ton?

3. What is the value of $\frac{7}{9}$ of a farm worth $2160?

4. A man had \$3300 in a bank, and took out $\frac{7}{11}$ of it; what had he left?

5. \$600 was divided equally among 20 persons; how much did 5 of them receive?

6. A lad had 60 eggs and sold $\frac{3}{5}$ of the lot at a cent apiece and the remainder at 2 cents apiece; how much money did he receive?

7. A grocer had 63 gallons of molasses. He sold $\frac{3}{7}$ of it at 40 cents a gallon and the rest at 50 cents a gallon; what did he get for the lot?

8. A boy had 40 oranges. He gave $\frac{3}{5}$ of them to his brother and $\frac{3}{8}$ of the remainder to his sister. How many had he left?

9. If I pay \$2.20 for half a yard of cloth, what will $\frac{5}{8}$ of a yd. come to?

10. A and B had a farm of 720 acres. They divide so that A takes $\frac{7}{12}$ and B $\frac{5}{12}$, how many acres did each get?

33. Fractions are usually divided into two classes—*Common* or *Vulgar Fractions* and *Decimal Fractions.*

34. A **Common** or **Vulgar Fraction** is one which is expressed by two numbers, one written above the other, with a line between them. $\frac{5}{8}$ is a vulgar fraction.

35. The number written below the line is called the **Denominator**. It indicates into how many equal parts the unit is divided.

36. The number written above the line is called the **Numerator**. It indicates how many of the equal parts, into which the unit is divided, are to be taken. Thus, in the expression $\frac{3}{5}$, the denominator, 5, denotes that a unit has been divided into 5 equal parts, called fifths; and the numerator, 3, indicates that 3 of these fifths are to be taken.

37. The numerator and denominator of a fraction are called **its terms.**

38. One of the equal parts into which a whole has been separated is called a **Fractional Unit.** $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{12}$ are fractional units.

REDUCTION OF FRACTIONS.

39. Reduction is the process of changing an expression to another of equal value.

Divide the line AB into 4 equal parts—AC, CD, DE and EB. Then AE is equal to $\frac{3}{4}$ of the line.

Now subdivide each of these equal parts into 3 equal parts, indicated by the shorter vertical lines. It will be seen that there are 12 of the smaller parts in the whole line and that each of them is $\frac{1}{12}$ of it. Now, 9 of them are equal to the distance AE, which is $\frac{3}{4}$ of the line.

Therefore $\frac{3}{4} = \frac{9}{12}$ or $\frac{9}{12} = \frac{3}{4}$.

But $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Therefore $\frac{3}{4} = \frac{3 \times 3}{4 \times 3}$

Again $\frac{9}{12} = \frac{3}{4}$

But $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

Therefore $\frac{9}{12} = \frac{9 \div 3}{12 \div 3}$

This shows that when the numerator and denominator of a fraction are each multiplied or divided by the same number, the value of the fraction is not changed.

CASE I.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

40. A fraction is in its lowest terms when the numerator and denominator are prime to each other, that is when they have no common factor.

EXAMPLE 9. Reduce $\frac{72}{84}$ to its lowest terms.

$$\frac{72}{84} = \frac{72 \div 12}{84 \div 12} = \frac{6}{7}$$

12 is the H. C F. of the terms of the fraction. By dividing both the terms by 12 we get $\frac{6}{7}$, which is equal in value to $\frac{72}{84}$. As 6 and 7 have no common factor the fraction is in its lowest terms.

The same result will be obtained by dividing by any factor common to both terms, and dividing the terms of the

result by any factor common to them, and continuing the process until the terms become prime to each other, *e.g.*:

$$\frac{72}{84} = \frac{24}{28}, \text{ dividing the terms by 3.}$$

$$\frac{24}{28} = \frac{6}{7}, \quad \text{“} \quad \text{“} \quad \text{“} \quad 4.$$

It will be observed that the product of the factors 3 and 4 is 12, which is the H. C. F. of the terms of the original fraction.

EXAMPLE 10. Reduce $\frac{108}{144}$ to its lowest terms.

$$\frac{108}{144} = \frac{9}{12} = \frac{3}{4}$$

Here we first divide the terms by 12 and the terms of the result by 3.

When the terms are large it is often convenient to find their H. C. F. and divide them by it:

$$\frac{285}{399} = \frac{285 \div 57}{399 \div 57} = \frac{5}{7}$$

EXERCISE IX.

Reduce to their lowest terms:

1.	$\frac{16}{20}$.	11.	$\frac{388}{404}$.	21.	$\frac{2424}{2712}$.
2.	$\frac{25}{35}$.	12.	$\frac{852}{1164}$.	22.	$\frac{361}{467}$.
3.	$\frac{18}{27}$.	13.	$\frac{1562}{2662}$.	23.	$\frac{5270}{6290}$
4.	$\frac{14}{21}$.	14.	$\frac{3100}{10925}$.	24.	$\frac{2418}{4797}$.
5.	$\frac{33}{44}$.	15.	$\frac{2288}{19800}$.	25.	$\frac{2755}{3515}$.
6.	$\frac{36}{108}$.	16.	$\frac{2640}{2970}$.	26.	$\frac{2103}{2253}$.
7.	$\frac{49}{168}$.	17.	$\frac{3650}{4380}$.	27.	$\frac{4055}{4985}$.
8.	$\frac{110}{132}$.	18.	$\frac{3944}{4812}$.	28.	$\frac{2654}{7271}$.
9.	$\frac{405}{900}$.	19.	$\frac{1991}{1320}$.	29.	$\frac{4555}{4955}$.
10.	$\frac{230}{690}$.	20.	$\frac{1670}{2105}$.	30.	$\frac{83000}{388400}$.

41. A **Mixed Number** consists of a whole number, and a fraction in the same expression; as $8\frac{3}{5}$.

42. A **Proper Fraction** is one whose numerator is less than its denominator; as $\frac{3}{7}$.

43. An **Improper Fraction** is one whose numerator is not less than its denominator; as $\frac{9}{7}$; $\frac{8}{8}$.

44. ORAL.

1. If I had three oranges, to how many boys could I give half of an orange?
2. To how many could I give a third?
3. How many halves in $2\frac{1}{2}$? in $3\frac{1}{2}$?
4. How many thirds in 2? in $2\frac{1}{3}$? in $2\frac{2}{3}$?
5. How many fourths in $3\frac{1}{4}$? in $3\frac{3}{4}$?
6. How many fifths in $3\frac{1}{5}$? in $2\frac{1}{5}$? in $7\frac{3}{5}$?

EXAMPLE 11. Reduce $8\frac{3}{5}$ to an improper fraction.

Since $1 = \frac{5}{5}$

$8 = \frac{40}{5}$

$8\frac{3}{5} = \frac{40}{5} + \frac{3}{5} = \frac{43}{5}$.

45. To reduce a mixed number to an improper fraction multiply the whole number by the denominator of the fraction and add the numerator to the product and put the denominator underneath, *e.g.*:

EXAMPLE 12. Reduce $5\frac{7}{8}$ to an improper fraction.

$$5\frac{7}{8} = \frac{5 \times 8 + 7}{8} = \frac{47}{8}$$

EXERCISE X.

Reduce to improper fractions:

1. $4\frac{3}{4}$.	8. $8\frac{9}{16}$.	15. $163\frac{169}{305}$.
2. $5\frac{3}{5}$.	9. $27\frac{11}{23}$.	16. $1614\frac{83}{91}$.
3. $6\frac{3}{8}$.	10. $31\frac{1}{100}$.	17. $2008\frac{100}{301}$.
4. $10\frac{5}{6}$.	11. $1\frac{99}{100}$.	18. $4096\frac{89}{360}$.
5. $11\frac{9}{11}$.	12. $163\frac{8}{9}$.	19. $1\frac{1114}{12178}$.
6. $1\frac{7}{8}$.	13. $12\frac{16}{21}$.	20. $41867\frac{37}{38}$.
7. $19\frac{10}{11}$.	14. $14\frac{37}{41}$.	21. $380\frac{1000}{90004}$.

22. Reduce 7 to sixteenths; 10 to fortieths.
23. Reduce 19 to twenty-firsts; 17 to elevenths.
24. Reduce 18 to seventy-seconds; 15 to fourteenths.

EXAMINATION PAPER, No. 2.

(TIME: ONE HOUR; VALUE OF EACH QUESTION: 10).

1. Two men set out from a certain town and travel in opposite directions. One travels 25 miles a day and the other 30. How far apart will they be in 10 days?

2. If 30 men can dig a drain in 20 days, how many men can dig it in 15 days?

3. A boy sold some strawberries. If he had received $\frac{1}{5}$ more than he did, he would have had $6.00. What did he get?

4. The sum of two numbers is 10000 and one of them is $\frac{7}{10}$ of 9990. What is the other?

5. If 400 bushels of potatoes cost $84.00, what will 250 bushels cost?

6. Paid $7.20 for 60 yards of calico. How many yards can I buy for $1.08?

7. Bought 60 acres of land at $32 per acre, and sold $\frac{2}{3}$ of it at $25 an acre, and the rest at $40 an acre. Did I gain or lose and how much?

8. What is the continued product of the prime factors of 12 and its other factors?

9. What is the largest number that will exactly divide 243162 and 816561?

10. James, John and William have the same amount of money. James' money is 10-cent pieces, John's is 25-cent pieces, and William's is 50-cent pieces. What is the least amount they can each have?

46. ORAL.

1. If a third of a yard of cloth will make a cap; how many yards will make 12 caps? 14 caps?

2. It takes a boy $\frac{1}{6}$ of an hour to write an exercise; how long would it take him to do 9 such exercises at the same rate?

3. If $\frac{1}{8}$ of a gallon of oil will fill one hall lamp, how many gallons will fill 10 lamps? 17? 19?

4. Paid $\frac{1}{8}$ of a dollar for 1 pound of cheese, what will 23 lbs. come to?

Example 13. Reduce $\frac{15}{4}$ to a mixed number:

$$\frac{15}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$
$$= 1 + 1 + 1 + \frac{3}{4}$$
$$= 3\frac{3}{4}.$$

It is evident that for every $\frac{4}{4}$ that is contained in the expression there is a unit. Now $\frac{4}{4}$ is contained 3 times in $\frac{15}{4}$ and there are $\frac{3}{4}$ over. Therefore $\frac{15}{4} = 3\frac{3}{4}$.

Example 14. Reduce $\frac{19}{7}$ to a mixed number:

$\frac{19}{7} = 19 \div 7 = 2\frac{5}{7}$.

47. To reduce an improper fraction to a mixed number, divide the numerator by the denominator.

REDUCTION OF FRACTIONS.

EXERCISE XI.

Reduce to whole or mixed numbers:

1.	$\frac{18}{5}$.	9.	$\frac{380}{120}$.	17.	$\frac{367804}{9876}$.
2.	$\frac{17}{8}$.	10.	$\frac{1920}{460}$.	18.	$\frac{84}{36}$.
3.	$\frac{20}{9}$.	11.	$\frac{1864}{23}$.	19.	$\frac{12700}{4006}$.
4.	$\frac{35}{6}$.	12.	$\frac{3807}{29}$.	20.	$\frac{32472}{3608}$.
5.	$\frac{42}{10}$.	13.	$\frac{13678}{468}$.	21.	$\frac{920920}{460}$.
6.	$\frac{84}{12}$.	14.	$\frac{93806}{240}$.	22.	$\frac{830768}{4973}$.
7.	$\frac{96}{24}$.	15.	$\frac{4257373}{373}$.	23.	$\frac{85678}{9387}$.
8.	$\frac{117}{96}$.	16.	$\frac{80678}{2033}$.	24.	$\frac{86107}{460}$.

CANCELLATION.

48 Cancellation is the process of shortening operations in division by dividing both divisor and dividend by factors common to both. The divisor is written under the dividend. Fractions may be reduced to their lowest terms by the same process.

Example 15.—Divide 24 by 16.

$$\frac{24}{16} = \frac{4 \times 2 \times 3}{4 \times 2 \times 2} = \frac{1 \times 1 \times 3}{1 \times 1 \times 2} = \frac{3}{2} = 1\frac{1}{2}$$

Here we divide the dividend 24 and the divisor 16 by the common factors 4 and 2.

Example 16.—Reduce $\frac{64}{96}$ to its lowest terms.

$$\frac{64}{96} = \frac{8 \times 4 \times 2}{8 \times 4 \times 3} = \frac{2}{3}$$

In this example, we simply reject the common factors 8 and 4 from numerator and denominator.

Example 17.—Divide the continued product of $6 \times 8 \times 21$ by $14 \times 12 \times 6$.

$$\frac{6 \times 8 \times 21}{14 \times 12 \times 6} = \frac{\overset{1}{\cancel{6}} \times \overset{2}{\cancel{8}} \times \overset{3}{\cancel{21}}}{\underset{2}{\cancel{14}} \times \underset{3}{\cancel{12}} \times \cancel{6}} = \frac{1 \times 1 \times 1}{1 \times 1 \times 1} = 1$$

Beginning with 6, the first figure in the upper line, we find it cancels out 6 in the lower line. We find that the next figure, 8, and 12 in the lower line, have a common factor, 4. We divide the 8 and 12 by this 4, placing the quotient 2 above the 8 and the quotient 3 below the 12. We next find 7 to be a common factor of 21 and 14. Dividing 21 and 14 by 7, we get the quotients 3 and 2 respectively. We now have quotients 2 and 3 in the upper line, and the same in the lower line. These quotients cancel out, leaving unity in the upper line and lower line, giving 1 as the value of the expression.

EXERCISE XII.

1. Divide $18 \times 6 \times 4 \times 42$ by $4 \times 9 \times 3 \times 7 \times 6$.

2. " $21 \times 8 \times 60 \times 8 \times 6$ by $7 \times 12 \times 3 \times 8 \times 3$.

3. " $16 \times 15 \times 14 \times 40 \times 16 \times 60 \times 50$ by $40 \times 24 \times 50 \times 20 \times 7 \times 10$.

4. How many cords of wood, at $4 a cord, must be given for 3 tons of hay, at $12 a ton?

5. How many firkins of butter, each holding 56 lbs., at 15 cents a pound, must be given for 8 barrels of loaf sugar, each containing 195 lbs., at 7 cents a pound?

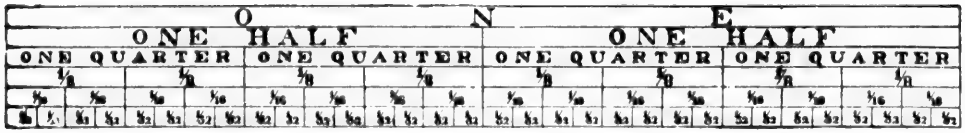
6. A farmer exchanged 39 bags of oats, each containing 2 bushels, at 35 cents a bushel, for 35 barrels of potatoes, each barrel holding 3 bushels. How much a bushel were the potatoes worth?

7. A man sold 6 loads of potatoes, each containing 20 bags and each bag 2 bushels. He sold them at 22 cents a bushel, and received in payment 4 boxes of tea, each containing 44 lbs. What was the tea worth a pound?

8. How many yards of dress material, at 24 cents a yard, may be bought for 36 lbs. of butter, at 18 cents a pound?

9. Divide 120960 by 1728, using factors.

10. Divide the continued product of 16, 14, 21, 28 and 12 by the continued product of 7, 5, 6, 8 and 42.



1. $\frac{1}{2}$ = how many fourths; eighths, &c.?
2. $\frac{1}{4}$ = how many eighths, &c.?
3. $\frac{3}{4}$ = how many sixteenths, &c.
4. Reduce $\frac{1}{8}$ to sixteenths; to thirty-seconds.
5. " $\frac{3}{8}$ " " "
6. " $\frac{5}{8}$ " " "

7. Construct a diagram, similar to the above, for thirds, sixths, twelfths, &c.

To reduce a fraction to an equivalent one with a different denominator.

EXAMPLE 18. Reduce $\frac{2}{3}$ to a fraction having 9 for its denominator.

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$ ANS.

49. Divide the denominator of the required fraction by the denominator of the fraction itself. Then multiply both numerator and denominator of the fraction by the quotient, and the result will be the equivalent fraction required.

EXAMPLE 19. Reduce $\frac{3}{8}$ to a fraction having 24 for its denominator.

$24 \div 8 = 3$.

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$ ANS.

EXAMPLE 20. Reduce 7 to ninths.

$$7 = \frac{7}{1} = \frac{7 \times 9}{1 \times 9} = \frac{63}{9} \quad \text{OR} \quad 1 = \frac{9}{9} \qquad 7 = \frac{9 \times 7}{9} = \frac{63}{9}$$

EXERCISE XIII.

Reduce.

1. $\frac{5}{6}$ to a fraction having 12 for its denominator.
2. $\frac{3}{4}$ " " 16 " "
3. $\frac{5}{8}$ " " 32 " "
4. $\frac{9}{12}$ " " 36 " "
5. $\frac{7}{10}$ " " 40 " "
6. $\frac{7}{12}$ " " 72 " "
7. $\frac{3}{14}$ " " 42 " "
8. $\frac{9}{16}$ " " 48 " "
9. $\frac{10}{13}$ " " 39 " "
10. $\frac{11}{20}$ " " 100 " "
11. $\frac{8}{21}$ " " 84 " "
12. $\frac{11}{24}$ " " 96 " "
13. $\frac{7}{18}$ " " 144 " "
14. $\frac{3}{23}$ " " 207 " "
15. $\frac{161}{204}$ " " 18360 " "

To reduce fractions to equivalent ones with a common denominator, the denominator being the least possible common to all the fractions.

EXAMPLE 21. Reduce $\frac{5}{8}$ and $\frac{7}{12}$ to their least common denominator.

L. C. M. of 8 and 12 = 24.

$\frac{5}{8} = \frac{15}{24}$. $\frac{7}{12} = \frac{14}{24}$.

50. Find the L. C. M. of all the denominators and reduce each fraction to an equivalent one having the L. C. M. for its denominator.

NOTE.—Mixed numbers may be reduced to improper fractions.

EXERCISE XIV.

Reduce to their least common denominator.

1. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$.
2. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{12}$.
3. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{11}{15}$.
4. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{7}{18}$.
5. $\frac{7}{18}$, $\frac{19}{24}$, $\frac{13}{30}$.
6. $\frac{3}{11}$, $\frac{7}{22}$, $\frac{19}{33}$.
7. $\frac{1}{17}$, $\frac{13}{51}$, $\frac{1}{2}$.
8. $\frac{5}{13}$, $\frac{3}{26}$, $\frac{7}{39}$.
9. $\frac{3}{8}$, $\frac{7}{10}$, $\frac{5}{12}$, $\frac{11}{18}$.
10. $\frac{4}{15}$, $\frac{7}{18}$, $\frac{17}{24}$, $\frac{19}{36}$.
11. $\frac{3}{14}$, $\frac{17}{21}$, $\frac{1}{28}$, $\frac{7}{56}$.
12. $\frac{3}{16}$, $\frac{23}{72}$, 9.
13. $\frac{17}{38}$, $\frac{11}{51}$, $\frac{3}{5}$.
14. 6, $6\frac{3}{4}$, 7.
15. $5\frac{1}{2}$, $2\frac{2}{5}$, $3\frac{1}{3}$.
16. $\frac{3}{57}$, $\frac{41}{133}$, $\frac{19}{209}$.
17. $\frac{7}{17}$, $\frac{9}{34}$, $\frac{23}{85}$.
18. $\frac{17}{306}$, $\frac{19}{301}$.

51. If two fractions have a common denominator the greater is that which has the greatest numerator.

Thus, $\frac{5}{8}$ is greater than $\frac{4}{8}$.

52. If two fractions have a common numerator the greater is that which has the smaller denominator.

Thus, $\frac{7}{8}$ is greater than $\frac{7}{9}$.

EXAMPLE 22. Arrange $\frac{3}{4}$, $\frac{11}{12}$ and $\frac{1}{6}$ in the increasing order of magnitude

$\frac{3}{4} = \frac{9}{12}$.
$\frac{11}{12} = \frac{11}{12}$.
$\frac{1}{6} = \frac{2}{12}$. Therefore $\frac{1}{6}$, $\frac{3}{4}$, $\frac{11}{12}$ is the order required.

EXERCISE XV.

Arrange in the increasing order of magnitude:

1. $\frac{3}{8}$, $\frac{7}{12}$, $\frac{5}{9}$, $\frac{3}{18}$.
2. $\frac{8}{11}$, $\frac{9}{12}$, $\frac{9}{14}$, $\frac{2}{9}$.
3. $\frac{5}{12}$, $\frac{7}{18}$, $\frac{11}{25}$, $\frac{13}{30}$.
4. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{11}{20}$, $\frac{13}{24}$.
5. $\frac{7}{24}$, $\frac{13}{48}$, $\frac{17}{64}$, $\frac{4}{15}$.
6. $\frac{17}{23}$, $\frac{16}{24}$, $\frac{14}{21}$, $\frac{7}{19}$.

53. ORAL.

1. John paid $\frac{2}{5}$ of a dollar for an arithmetic, $\frac{1}{5}$ for a slate, and $\frac{1}{5}$ for a reader. How much money did he spend?

2. James had a piece of string $\frac{1}{2}$ yard long and John had a piece $\frac{1}{4}$ of a yard long. What would be the length of both together?

3. How many eighths in $\frac{3}{4}$?

4. How many eighths in $\frac{3}{4} + \frac{3}{8}$?

5. Reduce the answer to a mixed number.

ADDITION OF FRACTIONS.

54. When fractions have a common denominator they can be added like whole numbers, thus $\frac{3}{8} + \frac{5}{8} + \frac{7}{8} = \frac{15}{8} = 1\frac{7}{8}$.

55. When fractions having different denominators are to be added, they must be reduced to equivalent ones having a common denominator.

EXAMPLE 23. Add $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.

L. C. M. of 4, 6 and 8 = 24.

$\frac{3}{4} + \frac{5}{6} + \frac{7}{8} = \frac{18}{24} + \frac{20}{24} + \frac{21}{24} = \frac{59}{24} = 2\frac{11}{24}$.

EXERCISE XVI.

Add the following:

1. $\frac{3}{8}$, $\frac{7}{12}$.
2. $\frac{5}{8}$, $\frac{3}{8}$, $\frac{11}{12}$.
3. $\frac{3}{7}$, $\frac{5}{14}$, $\frac{4}{21}$.
4. $\frac{7}{12}$, $\frac{5}{18}$, $\frac{8}{9}$.
5. $\frac{9}{10}$, $\frac{7}{12}$, $\frac{8}{15}$.
6. $\frac{3}{16}$, $\frac{17}{20}$, $\frac{5}{8}$.
7. $\frac{1}{41}$, $\frac{11}{21}$, $\frac{17}{63}$.
8. $\frac{15}{42}$, $\frac{19}{43}$.
9. $\frac{3}{5}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{16}{20}$.
10. $\frac{19}{20}$, $\frac{24}{25}$, $\frac{13}{30}$.
11. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.
12. $\frac{7}{8}$, $\frac{8}{9}$, $\frac{7}{18}$, $\frac{9}{12}$.
13. $\frac{7}{8}$, $\frac{5}{11}$, $\frac{13}{22}$.
14. $\frac{5}{16}$, $\frac{7}{24}$, $\frac{31}{36}$.
15. $\frac{7}{9}$, $\frac{8}{11}$, $\frac{5}{12}$.
16. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
17. $\frac{1}{11}$, $\frac{1}{13}$, $\frac{1}{15}$.
18. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{10}$.

To add mixed numbers:

EXAMPLE 24. Add $3\frac{3}{4}$, $5\frac{5}{6}$, $11\frac{7}{8}$.

$$3\tfrac{3}{4}+5\tfrac{5}{6}+11\tfrac{7}{8}$$
$$=3+5+11+\tfrac{3}{4}+\tfrac{5}{6}+\tfrac{7}{8}$$
$$=19+\tfrac{18}{24}+\tfrac{20}{24}+\tfrac{21}{24}$$
$$=19+\tfrac{59}{24}=19+2\tfrac{11}{24}$$
$$=21\tfrac{11}{24}\ \text{ANS.}$$

EXERCISE XVII.

Add the following:

1. $3\frac{1}{2}$, $4\frac{3}{4}$, $5\frac{5}{6}$.
2. $4\frac{5}{6}$, $7\frac{1}{8}$, $1\frac{7}{12}$.
3. $9\frac{3}{8}$, $10\frac{5}{12}$, $3\frac{5}{6}$.
4. $4\frac{3}{5}$, $6\frac{7}{10}$, $\frac{8}{15}$.
5. $9\frac{3}{10}$, $17\frac{13}{15}$, $8\frac{1}{20}$.
6. $8\frac{3}{8}$, $\frac{15}{16}$, $\frac{9}{24}$.
7. $4\frac{3}{11}$, $\frac{5}{12}$, $\frac{7}{22}$.
8. $6\frac{5}{17}$, $8\frac{18}{19}$, 17.
9. 64, 38, $\frac{5}{19}$.
10. $\frac{11}{19}$, 100, 60, $\frac{1}{4}$.
11. $161\frac{160}{161}$, $\frac{3}{322}$.
12. $108\frac{307}{1190}$, $490\frac{18}{19}$.
13. $36\frac{18}{23}$, $14\frac{11}{24}$, $\frac{7}{12}$.
14. $6\frac{6}{7}$, $7\frac{7}{8}$, $8\frac{8}{9}$, $9\frac{9}{10}$.
15. $5\frac{1}{2}$, $6\frac{3}{4}$, $3\frac{2}{17}$, $\frac{11}{29}$.
16. 18, $\frac{3}{11}$, $\frac{4}{22}$.
17. $18\frac{3}{8}$, 19, $\frac{3}{16}$.
18. 3, $3\frac{3}{19}$, $\frac{1}{17}$, 24.

56. ORAL.

1. A lad had $\frac{3}{4}$ of a dollar and gave one-fourth to his sister. How many fourths had he left?

2. A girl paid $\frac{7}{10}$ of a dollar for a geography, and $\frac{3}{10}$ for a grammar. How much more did she pay for the geography than for the grammar?

3. A girl had $\frac{5}{8}$ of a yard of silk. She made $\frac{3}{8}$ of it into a scarf. How many eighths had she left? How many quarters had she left?

SUBTRACTION OF FRACTIONS.

57. A fraction may be subtracted from another having the same denominator by subtracting the numerator of the one from that of the other; thus

$\frac{9}{10} - \frac{8}{10} = \frac{1}{10}$; $\frac{11}{12} - \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$.

To subtract fractions having different denominators.

EXAMPLE 25. From $\frac{5}{6}$ take $\frac{3}{4}$.

$\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$.

58. When necessary reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from that of the minuend, and place the difference over the common denominator.

EXERCISE XVIII.

What is the value of:

1. $\frac{3}{4} - \frac{1}{2}$.
2. $\frac{5}{6} - \frac{3}{4}$.
3. $\frac{5}{8} - \frac{1}{6}$.
4. $\frac{11}{12} - \frac{7}{8}$.
5. $\frac{13}{18} - \frac{10}{21}$.
6. $\frac{9}{16} - \frac{11}{24}$.
7. $\frac{17}{18} - \frac{19}{24}$.
8. $\frac{5}{13} - \frac{3}{17}$.
9. $\frac{16}{23} - \frac{1}{11}$.
10. $\frac{9}{10} - \frac{17}{30}$.
11. $\frac{1}{4} - \frac{1}{8}$.
12. $\frac{3}{14} - \frac{5}{42}$.
13. $\frac{11}{12} - \frac{11}{13}$.
14. $\frac{18}{19} - \frac{5}{7}$.
15. $\frac{37}{30} - \frac{29}{35}$.
16. $\frac{97}{100} - \frac{67}{150}$.
17. $\frac{33}{34} - \frac{54}{102}$.
18. $\frac{100}{101} - \frac{63}{99}$.

EXAMPLE 26. From $6\frac{3}{4}$ take $5\frac{1}{6}$.

$6\frac{3}{4} = 6\frac{18}{24}$.
$5\frac{1}{6} = 5\frac{4}{24}$.
$1\frac{14}{24} = 1\frac{7}{12}$ Ans.

EXAMPLE 27. From $5\frac{3}{8}$ take $3\frac{7}{12}$.

$5\frac{3}{8} = 5\frac{9}{24} = 4 + 1\frac{9}{24} = 4\frac{33}{24}$.
$3\frac{7}{12} = 3\frac{14}{24}$.
$1\frac{19}{24}$ Ans.

EXAMPLE 28. From 7 take $3\frac{5}{6}$.

$7 = 6\frac{6}{6}$.
$3\frac{5}{6} = 3\frac{5}{6}$
$3\frac{1}{6}$ Ans.

EXERCISE XIX.

What is the value of:

1. $6\frac{3}{8} - 4\frac{5}{8}$.
2. $5\frac{3}{4} - 3\frac{7}{8}$.
3. $10\frac{5}{9} - 7\frac{11}{12}$.
4. $10\frac{7}{10} - 8\frac{7}{9}$.
5. $12\frac{3}{8} - \frac{5}{9}$.
6. $8\frac{3}{11} - \frac{7}{12}$.
7. $16 - 8\frac{3}{14}$.
8. $17\frac{9}{16} - 11\frac{17}{18}$.
9. $20\frac{1}{4} - \frac{41}{43}$.
10. $170\frac{5}{13} - 169\frac{13}{14}$.
11. $16\frac{3}{8} - 14$.
12. $8\frac{3}{100} - 4\frac{79}{99}$.
13. $17 - \frac{9}{16}$.
14. $16\frac{3}{19} - \frac{11}{17}$.
15. $1\frac{8}{9} - \frac{9}{10}$.
16. $1\frac{99}{100} - \frac{100}{101}$.
17. $106\frac{7}{101} - 1$.
18. $84\frac{17}{69} - 4\frac{110}{115}$.
19. $63\frac{8}{103} - 14\frac{9}{104}$.
20. $164 - \frac{1}{161}$.
21. $180\frac{163}{400} - 75$.
22. $16\frac{3}{115} - \frac{11}{46}$.
23. $17\frac{3}{200} - 16\frac{3}{200}$.
24. $1 - \frac{19}{1000}$.
25. $131\frac{130}{4000} - \frac{9900}{90000}$.
26. $1600\frac{63}{107} - \frac{64}{107}$.
27. $8\frac{3}{14} - 7\frac{97}{98}$.
28. $\frac{1}{117} - \frac{1}{319}$.
29. $\frac{1}{117} - \frac{3}{429}$.
30. $1\frac{3}{927} - \frac{3}{721}$.

59. Brackets may be conveniently used in working some questions, *e.g.*:

From a large tub of butter, weighing $28\frac{1}{8}$ lbs. I filled 2 tubs, weighing $5\frac{1}{4}$ and $6\frac{1}{2}$ lbs. respectively. How many pounds were left?

EXAMPLE 29.—$28\frac{1}{8} - (5\frac{1}{4} + 6\frac{1}{2})$
$= 28\frac{1}{8} - 11\frac{3}{4}$
$= 28\frac{1}{8} - 11\frac{6}{8} = 16\frac{3}{8}$ lbs.

EXERCISE XX.

1. $6\frac{3}{8} - (\frac{2}{3} + \frac{5}{6})$.
2. $(17\frac{1}{2} + 3\frac{1}{4}) - (\frac{3}{8} + 1\frac{4}{15})$.
3. $(41\frac{1}{2} + 5\frac{1}{7}) - (\frac{1}{3} - \frac{1}{7})$.
4. $(14\frac{3}{4} - \frac{7}{8}) - (3\frac{1}{8} - 2\frac{1}{4})$.
5. $6\frac{5}{8} - (3 + 1\frac{1}{5}) + \frac{3}{7}$.
6. $7\frac{1}{2} - 1\frac{1}{3} - (5\frac{1}{3} - \frac{5}{6})$.

EXERCISE XXI.

1. Out of a tub of butter, weighing $5\frac{3}{8}$ lbs., I took $2\frac{3}{4}$ lbs. How much was left?

2. The larger of 2 pigs weighed $236\frac{7}{12}$ lbs. and the smaller $20\frac{5}{8}$ lbs. less. What did the smaller weigh?

3. From a barrel of water, containing $29\frac{3}{7}$ gallons, $16\frac{7}{11}$ gallons were drawn. How many gallons were left?

4. James and John have $\$14\frac{6}{10}$ between them. James' share is $\$3\frac{5}{8}$. What is John's?

5. A and B each own a farm. There are $160\frac{11}{16}$ acres in A's, which is $15\frac{7}{8}$ acres more than B's. How many acres in B's.

6. To what fraction must I add $3\frac{7}{9}$ so that the sum will be 18?

7. The sum of $1\frac{3}{5}$ and 6, added to a certain fraction, makes $10\frac{1}{5}$. What is the fraction? (Use brackets in this and the next two questions).

8. A man who had \$100 gave $\$7\frac{3}{8}$ to his eldest son, $\$5\frac{3}{5}$ to his second son, $\$5\frac{5}{8}$ to his daughter and the remainder to his wife. How much did his wife get?

9. From the sum of $3\frac{3}{8}$, $\frac{5}{9}$, $7\frac{7}{11}$, take the difference of 10 and $\frac{1}{3\frac{1}{1}}$.

10. A has $\frac{3}{4}$ of a dollar more than B, who has $\$2\frac{3}{5}$. How much have A and B together?

EXAMINATION PAPER. No. 3.

(TIME: ONE HOUR. VALUE OF EACH QUESTION: 10).

1. Reduce $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{18}$ and $\frac{11}{36}$ to their least common denominator.

2. From the sum of $6\frac{3}{8}$ and $1\frac{7}{12}$ take their difference.

3. Find the value of $\frac{63 \times 49 \times 54 \times 21 \times 11}{66 \times 3 \times 56 \times 42 \times 28}$ by cancellation.

4. The distance from Halifax to Montreal by rail is 756 miles. If a train travelling at the rate of 35 miles an hour leaves Halifax at the same time that a freight train travelling at the rate of 20 miles an hour leaves Montreal, how long will it be until they meet?

5. If 10 pounds of butter are worth 15 pounds of cheese, and 25 pounds of cheese are worth \$2.75, what is butter worth a pound?

6. A man after paying $\frac{3}{5}$ of his money for a horse, had left \$80 less than what he had paid for the horse. How much money had he at first?

7. A farmer paid $\frac{3}{8}$ of his money for stock, $\frac{2}{7}$ for a new barn, and had \$190 left. How much had he at first?

8. How long will it take 18 men to do the same amount of work that 12 men can do in 30 days?

9. $\frac{2}{3}$ of A's money is equal to $\frac{3}{7}$ of B's, and B has \$210. How much money has A?

10. To how many persons can I give $\frac{1}{30}$ of a barrel of flour if I have only $\frac{5}{6}$ of a barrel?

60. A fraction may be looked upon as one or more equal parts of a unit, or its numerator may be regarded as a dividend and its denominator as a divisor. Thus $\frac{3}{4}$ may be conceived as representing 3 equal parts of a unit when the unit is divided into 4 equal parts, or as 3 whole units divided by 4. Thus $\frac{3}{4}$ of 1 dollar = 3 dollars divided by 4. $\frac{3}{4}$ of 1 foot = $\frac{1}{4}$ of 3 feet.

61. ORAL.

1. What will 8 yards cost at \$$\frac{1}{4}$ a yard? What will 10 cost?

2. What will 10 yards cost at \$$\frac{3}{4}$ a yard? What will 12 cost?

3. How many yards will it take to make 3 waistcoats if $\frac{1}{8}$ of a yard make one?

4. A man can chop $\frac{5}{6}$ of a cord of wood in a day; how much can he chop in 10 days?

5. If a man earn $\frac{7}{8}$ of a dollar in 1 day, how many dollars will he earn in 12 days?

EXAMPLE 30. Multiply $\frac{3}{5}$ by 4.

$\frac{3}{5} \times 4 = \frac{12}{5} = 2\frac{2}{5}$.

EXAMPLE 31. Multiply $\frac{5}{12}$ by 6.

$\frac{5}{12} \times 6 = \frac{30}{12} = \frac{5 \times 6}{12} = \frac{5}{2} = 2\frac{1}{2}$.

62. To multiply a fraction by a whole number multiply the numerator of the fraction by the number for a new numerator and place the denominator under the result; or when possible divide the denominator of the fraction by the number.

In the second example

$$\frac{5}{12} \times 6 = \frac{5 \times 6}{12} = \frac{5}{2} \quad \text{or} \quad \frac{5}{12} \times 6 = \frac{5}{12 \div 6} = \frac{5}{2}$$

63. Mixed numbers may be reduced to improper fractions before being multiplied.

EXAMPLE 32. Multiply $3\frac{3}{5}$ by 8.

$8\frac{3}{5} \times 8 = \frac{43}{5} \times 8 = \frac{344}{5} = 68\frac{4}{5}$.

EXERCISE XXII.

Find the value of (by both methods when possible):

1. $\frac{3}{4} \times 5$.	**7.** $8 \times \frac{3}{8}$.	**13.** $3\frac{10}{11} \times 7$.
2. $\frac{3}{8} \times 4$.	**8.** $16 \times \frac{3}{4}$.	**14.** $18 \times 2\frac{3}{4}$.
3. $\frac{3}{5} \times 8$.	**9.** $\frac{3}{11} \times 19$.	**15.** $18\frac{9}{27} \times 540$.
4. $\frac{5}{12} \times 10$.	**10.** $17 \times \frac{5}{34}$.	**16.** $6\frac{11}{19} \times 57$.
5. $\frac{5}{9} \times 21$.	**11.** $1\frac{1}{2} \times 6$.	**17.** $2\frac{3}{8} \times 100$.
6. $\frac{5}{11} \times 18$.	**12.** $7\frac{3}{8} \times 16$.	**18.** $1000 \times \frac{3}{125}$.

TO DIVIDE A FRACTION BY A WHOLE NUMBER:

EXAMPLE 33. Divide $\frac{8}{9}$ by 4.

8 ninths $\div 4 = 2$ ninths $= \frac{2}{9}$.

But $\frac{2}{9} = \frac{2 \times 4}{9 \times 4} = \frac{8}{36}$.

Therefore $\frac{8}{9}$ divided by $4 = \frac{8}{9 \times 4} = \frac{8}{36} = \frac{2}{9}$.

64. To divide a fraction by a whole number divide the numerator by the whole number, or multiply the denominator by it. When the numerator is not a multiple of the whole number, multiply the denominator by it.

EXAMPLE 34. Divide $\frac{6}{7}$ by 3:

$\frac{6}{7} \div 3 = \frac{2}{7}$.

Or $\frac{6}{7} \div 3 = \frac{6}{7 \times 3} = \frac{2}{7}$.

EXERCISE XXIII.

Divide:

1. $\frac{8}{9} \div 4$.	**4.** $\frac{18}{19} \div 6$.	**7.** $4\frac{3}{4} \div 10$.
2. $\frac{7}{10} \div 6$.	**5.** $\frac{18}{23} \div 5$.	**8.** $7\frac{3}{5} \div 19$.
3. $\frac{11}{12} \div 8$.	**6.** $\frac{18}{19} \div 18$.	

EXAMPLE 35. Multiply $\frac{5}{7}$ by $\frac{3}{4}$:

$\frac{5}{7} \times 3 = \frac{15}{7}$ which is 4 times too much since 3 is 4 times $\frac{3}{4}$. We therefore must divide $\frac{15}{7}$ by 4 to get the true answer:

$$\frac{15}{7} \div 4 = \frac{15}{7 \times 4} = \frac{5 \times 3}{7 \times 4}.$$

$$= \frac{\text{product of numerators.}}{\text{product of denominators.}}$$

65. Multiply all the numerators together for a new numerator and all the denominators together for a new denominator.

Mixed numbers are reduced to improper fractions before being multiplied.

66. A whole number may be put in the form of a fraction by putting one for its denominator; thus:

$$\frac{3}{8} \times 6 = \frac{3}{8} \times \frac{6}{1} = \frac{3 \times 6}{8 \times 1} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}.$$

EXERCISE XXIV.

Simplify, using the method of cancellation:

1. $\frac{3}{8} \times \frac{3}{4}$.
2. $\frac{5}{8} \times \frac{3}{7}$.
3. $\frac{5}{6} \times \frac{7}{8}$.
4. $\frac{7}{9} \times \frac{8}{9}$.
5. $\frac{9}{10} \times \frac{3}{7}$.
6. $\frac{4}{17} \times \frac{9}{20}$.
7. $6\frac{3}{4} \times 7$.
8. $5\frac{3}{8} \times 6\frac{3}{10}$.
9. $2\frac{3}{4} \times 1\frac{1}{22} \times \frac{1}{23}$.
10. $3\frac{1}{4} \times 1\frac{1}{3} \times \frac{1}{26}$.
11. $4\frac{3}{4} \times 4 \times \frac{7}{38}$.
12. $8\frac{3}{8} \times 3\frac{3}{7} \times 2$.
13. $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times 2$.
14. $1\frac{1}{4} \times \frac{8}{9} \times \frac{9}{16} \times 1\frac{3}{5}$.
15. $3\frac{1}{4} \times \frac{5}{69} \times 23\frac{4}{17}$.
16. $\frac{18}{25} \times 1\frac{1}{14} \times \frac{7}{36}$.
17. $10\frac{3}{11} \times \frac{5}{51} \times \frac{17}{252}$.
18. $\frac{19}{20} \times 1\frac{1}{33} \times \frac{3}{57}$.

67. A **Simple Fraction** is one in which both numerator and denominator are whole numbers, as $\frac{3}{4}$.

68. A **Compound Fraction** is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{3}{4}$.

Since $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3}$ of $\frac{9}{12}$, Therefore $\frac{2}{3}$ of $\frac{9}{12} = \frac{6}{12}$;

And $\frac{1}{3}$ of $\frac{9}{12} = \frac{3}{12}$, That is, $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{2}{3} \times \frac{3}{4}$.

From this it appears that "of" and the symbol × mean the same thing. There is one case, however, in which a distinction is made between "of" and the sign of multiplication. It will be explained further on.

EXERCISE XXV.

Simplify:

1. $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{4}{5}$.
2. $\frac{3}{16}$ of $1\frac{1}{8} \times \frac{7}{18}$.
3. $2\frac{3}{4}$ of $\frac{3}{\frac{1}{2}}$ of 12.
4. $\frac{3}{7}$ of $21 \times \frac{1}{5}$.
5. $\frac{2}{3}$ of $\frac{3}{4}$ of 2.
6. $\frac{5}{9}$ of $3\frac{1}{4}$ of 26.
7. $\frac{4}{5} \times \frac{2}{3}$ of 63.
8. $\frac{5}{9}$ of $\frac{6}{20}$ of $\frac{100}{101}$.
9. $\frac{5}{11}$ of $\frac{22}{25}$ of $\frac{17}{77}$.
10. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of 5.

EXAMINATION PAPER. No. 4.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10).

1. How many barrels of flour at $3.60 can I buy for $12\frac{2}{3}$ tons of hay @ $10.80?

2. Divide the L. C. M. of 24, 36 and 48 by their G. C. D.

3. Multiply the sum of $\frac{5}{8}$ and $\frac{7}{12}$ by their difference.

4. A man by mistake gave $\frac{1}{5}$ of a certain sum of money for an article instead of $\frac{1}{6}$ and thus gave $2.40 too much. What was the sum of money?

5. A grocer bought 60 lbs. of tea @ 20c. a pound. At how much per pound must he sell it to gain $6 on the lot?

6. An artisan earns $9\frac{3}{5}$ a week and spends $6\frac{7}{8}$. How much can he save in a year?

7. Add $3\frac{3}{16}$, $7\frac{7}{34}$, $\frac{19}{30}$.

8. From $1\frac{11}{12}$ take $\frac{12}{13}$.

9. If I pay $63.72 for 59 yards of Canadian tweed, how many yards can I buy for $191.16?

10 A can do a piece of work in 3 days and B can do the same work in 4 days. If they work together how much of the work will they do in 1 day?

EXERCISE XXVI.

1. At $3\frac{3}{4}$ cents a pound what will 20 lbs. of brown sugar cost?

2. At $5\frac{1}{4}$ cents a pound what will be the cost of $10\frac{2}{5}$ of granulated?

3. What is the value of a tub of butter, weighing $13\frac{3}{8}$ lbs., at $21\frac{1}{2}$ cents?

4. A man earns $\$2\frac{3}{8}$ a day, how much will he earn in $7\frac{1}{2}$ days?

5. What is the value of $17\frac{3}{4}$ lbs. of tea at $\$\frac{9}{20}$ per lb?

6. How much will $6\frac{5}{8}$ yds. of tweed come to at \$1.90 a yard?

7. If a man can mow $\frac{7}{8}$ of an acre in a day, how much will he mow in $\frac{5}{8}$ of a day?

8. A man can chop and pile $\frac{9}{10}$ of a cord of wood in 1 day; what will he do in $\frac{3}{4}$ of a day?

9. Paid $\$\frac{1}{5}$ per barrel for flour and sold the lot at \$5 per barrel. How much did I gain on 20 barrels?

10. $\frac{3}{8}$ of a dollar was $\frac{1}{5}$ the cost of a pair of skates: what was the whole cost?

11. $\frac{3}{5}$ of a yd. of silk will make a handkerchief. How many yards will make 20 handkerchiefs?

12. A farmer in selling oats used a box for a bushel measure. He afterwards found that the box was $\frac{1}{10}$ of a bushel too large. If oats were worth 35c. a bushel, how much did he lose when he sold a box full for a bushel?

TO DIVIDE A WHOLE NUMBER OR A FRACTION BY A FRACTION.

EXAMPLE 36. Divide 7 by $\frac{5}{8}$.

$7 \div \frac{5}{8} = \frac{56}{8} \div \frac{5}{8} =$

$\frac{56 \text{ eighths}}{5 \text{ eighths}} = \frac{56}{5} = \frac{7 \times 8}{5} = 7 \times \frac{8}{5}$ = dividend 7, × by divisor, $\frac{5}{8}$, inverted.

EXAMPLE 37. Divide $\frac{3}{5}$ by $\frac{7}{8}$.

$\frac{3}{5} \div \frac{7}{8} = \frac{24}{40} \div \frac{35}{40} =$

$\frac{24 \text{ fortieths}}{35 \text{ fortieths}} = \frac{24}{35} = \frac{3 \times 8}{5 \times 7} = \frac{3}{5} \times \frac{8}{7}$ = dividend, $\frac{3}{5}$, multiplied by the divisor, $\frac{7}{8}$, inverted.

69. Hence to divide one fraction by another we invert the divisor and multiply the dividend by it.

EXERCISE XXVII.

Divide:

1. $12 \div \frac{2}{3}$.
2. $14 \div \frac{7}{11}$.
3. $20 \div \frac{4}{5}$.
4. $3\frac{1}{2} \div \frac{2}{3}$.
5. $40 \div 3\frac{2}{5}$.
6. $\frac{15}{16} \div \frac{9}{10}$.
7. $8\frac{3}{4} \div 1\frac{1}{6}$.
8. $3\frac{3}{7} \div 1\frac{20}{21}$.
9. $80\frac{4}{5} \div \frac{17}{15}$.
10. $6\frac{3}{13} \div 1\frac{5}{39}$.
11. $8\frac{7}{10} \div \frac{19}{20}$.
12. $4\frac{1}{100} \div 2\frac{1}{100}$.
13. $4\frac{17}{20} \div 1\frac{1}{153}$.
14. $33\frac{1}{3} \div \frac{1}{50}$.
15. $4\frac{11}{18} \div 1\frac{5}{36}$.

16. How many boxes, each holding $\frac{5}{6}$ of a quart, can be filled with 10 quarts of berries?

17. How many pairs of trousers can be made from $27\frac{1}{2}$ yards of cloth, if $2\frac{1}{2}$ yards make 1 pair?

18. A boy gets $\$\frac{3}{4}$ a day to work at haymaking. In how many days will he earn \$15?

19. At $2\frac{2}{5}$ dollars a yard, how many yards can be bought for $\$50\frac{2}{5}$?

20. A man travels $4\frac{3}{7}$ miles an hour. In how many hours will he travel 62 miles?

21. If a bag holds $2\frac{3}{8}$ bushels, how many bags will hold 57 bushels?

22. How many lots, each containing $\frac{5}{9}$ of an acre, can be made from 90 acres?

23. The product of 2 numbers is $\frac{18}{19}$, and one of them is $\frac{6}{7}$. What is the other?

24. The product of 2 numbers is $16\frac{1}{8}$, and one of them is $5\frac{1}{4}$. What is the other?

25. If $5\frac{2}{7}$ bushels of oats cost $\$2\frac{3}{14}$, what will 1 bushel come to?

EXAMINATION PAPER. No. 5.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10).

1. Write in words 63000040019.

2. From seventeen trillion one thousand forty-one take ninety thousand nine hundred ninety-nine.

3. Find the prime factors of 64119.

4. What is the G. C. D. of 468963 and 596862?

5. From the product of $7\frac{5}{6}$ and $1\frac{1}{17}$ take their quotient.

6. A man's house and lot was worth \$3,300. If the house was worth ten times the lot, what were they each worth?

7. A lady gave a certain amount of money to the poor every month, always giving the same sum. When there were 10 persons, each person received $\$1\frac{1}{2}$ more than when there were 12. What was the sum distributed?

8. Divide \$630 among A, B and C, so as to give A $\frac{2}{7}$ of it, B $\frac{5}{8}$ of the remainder and C what was then left.

9. What is the average of $6\frac{3}{4}, 5\frac{2}{3}$, $1\frac{5}{6}$ and $2\frac{7}{8}$?

10. If 160 dozen of eggs cost \$17.60, how many boxes, each holding 20 dozen, can be bought for \$44.00?

EXAMPLE 38. If $\frac{3}{4}$ of a yard of diagonal is worth 90c., what is $\frac{5}{6}$ of a yard worth?

$\frac{3}{4} = \frac{9}{12}$

$\frac{5}{6} = \frac{10}{12}$

$\frac{9}{12}$ of a yard cost 90c.

$\frac{1}{12}$ " costs $\frac{90}{9}$c.

$\frac{10}{12}$ " cost $\frac{90\text{c.} \times 10}{9} = 100\text{c.} = \$1.00.$

EXERCISE XXVIII.

1. Paid \$3.60 for $\frac{4}{5}$ of a ton of coal, what will $\frac{7}{8}$ of a ton cost?

2. $\frac{3}{10}$ of a ton of hay is worth \$2.70; what is $\frac{5}{6}$ of a ton worth?

3. $\frac{3}{7}$ of a field grows 160 bushels of oats, what would $\frac{2}{3}$ of it grow?

4. A man sold $\frac{5}{11}$ of his farm for \$600, what is the whole of it worth?

5. A farmer raised oats on $\frac{2}{3}$ of a piece of land, and wheat on $\frac{1}{2}$ of the remainder; if what was then left was 3 acres, how many acres in the whole piece?

6. $\frac{3}{5}$ of a pole was broken off and $\frac{1}{4}$ of what was left measured 6 feet, what was the whole length?

7. If $\frac{7}{11}$ of an acre yields 21 bushels, what will $\frac{1}{8}$ of an acre yield?

8. If $\frac{3}{8}$ of a yard of cloth cost \$0.24, what would $2\frac{3}{4}$ yards cost?

9. Paid \$2.60 for $\frac{5}{9}$ of a barrel of flour, what is $1\frac{1}{4}$ barrels worth?

10. What is $\frac{2}{3}$ of a boat worth if $\frac{7}{9}$ of it is worth \$21.

EXAMPLE 39. If $\frac{3}{5}$ of a dollar buy 2 bushels of oats, what can I buy for $\frac{2}{3}$ of a dollar?

$\frac{3}{5} = \frac{9}{15}$
$\frac{2}{3} = \frac{10}{15}$

$\frac{9}{15}$ of a dollar buy 2 bushels
$\frac{1}{15}$ " buys $\frac{2}{9}$ "
$\frac{10}{15}$ " buy $\frac{2 \times 10}{9} = 2\frac{2}{9}$ bushels.

EXERCISE XXIX.

1. If $\frac{3}{4}$ of a dollar buys 48 eggs, how many can I buy for $\frac{5}{8}$ of a dollar?

2. $\frac{3}{14}$ of a log measures 12 feet; what is the length of $\frac{3}{20}$ of it?

3. Paid \$1440 for $\frac{5}{18}$ of a ship; what part can I buy for \$4320?

4. $\frac{9}{13}$ of a cheese weighs 27 lbs; what will $\frac{5}{39}$ of it weigh?

5. Gave $\frac{7}{8}$ of a ton of hay for 40 bushels of potatoes; how many bushels can I get for $\frac{1}{12}$ of a ton?

6. If $\frac{7}{20}$ of a lot of land sold for 30 head of cattle, what is $\frac{3}{4}$ of the lot worth if cattle be \$21 a head?

7. $\frac{2}{3}$ of $\frac{3}{5}$ of a dollar buys 3 doz. of eggs; how many dozen will $\frac{5}{8}$ of \$4.00 buy.

COMPLEX FRACTIONS.

70. A **Complex Fraction** is one that has a fraction, or mixed number, for its numerator or denominator or both; as,

$\frac{\frac{3}{4}}{6}$, $\frac{7}{\frac{3}{5}}$, $\frac{3\frac{1}{2}}{7\frac{1}{4}}$.

EXAMPLE 40. Reduce $\frac{\frac{3}{4}}{6}$ to a simple fraction:

$\frac{\frac{3}{4}}{6} = \frac{3}{4} \div 6 = \frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$. Or $\frac{\frac{3}{4}}{6} = \frac{\frac{3}{4} \times 4}{6 \times 4} = \frac{3}{24} = \frac{1}{8}$.

EXAMPLE 41. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{9}{10}$. Or $\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{\frac{3}{4} \times 12}{\frac{5}{6} \times 12} = \frac{9}{10}$.

It will be observed that in the second method of working the above examples, that both numerator and denominator are multiplied by the L. C. M. of the denominators of the fractional parts.

EXERCISE XXX.

Simplify by both methods:

1. $\frac{\frac{3}{4}}{\frac{5}{6}}$
2. $\frac{5}{\frac{3}{4}}$
3. $\frac{\frac{3}{4}}{6}$
4. $\frac{2\frac{1}{2}}{1\frac{1}{3}}$
5. $\frac{1\frac{1}{3}}{1\frac{1}{4}}$
6. $\frac{2\frac{1}{2}}{1\frac{2}{5}}$
7. $\frac{3\frac{2}{3}}{3\frac{3}{4}}$
8. $\frac{1\frac{2}{5}}{3\frac{1}{3}}$
9. $\frac{3\frac{3}{4}}{\frac{3}{5}}$
10. $\frac{6}{\frac{3}{15}}$
11. $\frac{2\frac{3}{5}}{1\frac{1}{4}}$
12. $\frac{5\frac{1}{6}}{2\frac{5}{8}}$
13. $\frac{7\frac{3}{8}}{3\frac{5}{12}}$
14. $\frac{1\frac{1}{7}}{1\frac{1}{5}}$
15. $\frac{\frac{1}{10}}{\frac{1}{11}}$

EXERCISE XXXI.

Simplify:

1. $\frac{6\frac{1}{2} + 4\frac{1}{4}}{6\frac{1}{2} - 4\frac{1}{4}}$
2. $\frac{8\frac{1}{4} - \frac{5}{6}}{8\frac{1}{4} + \frac{5}{6}}$
3. $\frac{\frac{2}{3} \text{ of } \frac{3}{8}}{4 - 3\frac{1}{4}}$
4. $\frac{\frac{3}{4} + \frac{5}{6}}{\frac{2}{3} \text{ of } 16}$
5. $\frac{8\frac{1}{4} - 4}{2 \times 3\frac{1}{4}}$
6. $\frac{\frac{2}{3} + \frac{5}{6}}{18}$
7. $\frac{\frac{3}{4} \div \frac{5}{6}}{\frac{3}{4} \times \frac{5}{6}}$
8. $\frac{\frac{3}{4} + \frac{5}{6} + \frac{7}{8}}{\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } 8}$
9. $\frac{\frac{3}{4} \times \frac{3}{4} \div 2\frac{1}{4}}{2\frac{1}{4} \times \frac{4}{9}}$

71. ORAL.

To find what part or fraction one number is of another.

1. 8 is what part of 24? 1 is $\frac{1}{24}$ of 24; 8 is $\frac{8}{24}$ of 24; or, $\frac{1}{3}$ of 24.
2. 3 is what part of 7? 5 of 9? 10 of 12? 17 of 20?
3. What part of 8 is 5? Of 10 is 7? Of 11 is 4? Of 30 is 20?
4. What fraction of 9 is 7? Of 16 is 9?
5. 10 is what fraction of 19? 8 is what fraction of 21?

6. What part of 5 is $\frac{3}{4}$? $5 = \frac{20}{4}$;
$\frac{1}{4}$ is $\frac{1}{20}$ of $\frac{20}{4}$; $\frac{3}{4}$ is $\frac{3}{20}$ of $\frac{20}{4}$.

7. What part of $\frac{5}{6}$ is $\frac{3}{4}$? $\frac{5}{6} = \frac{10}{12}$; $\frac{3}{4} = \frac{9}{12}$;
Since $\frac{1}{12}$ is $\frac{1}{10}$ of $\frac{1}{12}$, $\frac{9}{12}$ is $\frac{9}{10}$ of $\frac{10}{12}$.
That is, $\frac{9}{10}$ is $\frac{9}{10}$ of $\frac{5}{6}$.

72. It will be noticed that the number following the word "of" in such exercises as the above, may be put as the denominator, and the other number as the numerator, of the fraction required. Thus, in Exercises 6 and 7:

$$\frac{\frac{3}{4}}{5} = \frac{3}{20}; \quad \frac{\frac{3}{4}}{\frac{5}{6}} = \frac{9}{10}.$$

EXERCISE XXXII.

What part of

1. 16 is 9?
2. 18 is $\frac{5}{6}$?
3. 14 is $\frac{3}{10}$?
4. $\frac{2}{3}$ is $\frac{3}{9}$?
5. $4\frac{1}{2}$ is 3?
6. $8\frac{1}{4}$ is $2\frac{1}{16}$?
7. $7\frac{1}{9}$ is 5?
8. $18\frac{1}{4}$ is 6?
9. $3\frac{1}{2}$ is 1?
10. $10\frac{3}{4}$ is $5\frac{1}{8}$?

EXAMPLE 42. 160 is $\frac{5}{8}$ of what number

$\frac{5}{8}$ of the number $= 160$
$\frac{1}{8}$ " " $= \frac{160}{5}$
$\frac{8}{8}$ " " $= \frac{160 \times 8}{5} = 256.$

EXERCISE XXXIII.

Find the number of which

1. 120 is $\frac{3}{4}$.
2. 60 is $\frac{5}{8}$.
3. 90 is $\frac{10}{11}$.
4. 180 is $\frac{5}{6}$.
5. 81 is $\frac{9}{20}$.
6. 364 is $\frac{7}{18}$.
7. 630 is $\frac{9}{11}$.
8. $4\frac{1}{2}$ is $\frac{2}{3}$.
9. $7\frac{3}{4}$ is $\frac{3}{8}$.

EXAMINATION PAPER. No. 6.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10.)

1. A contractor after paying \$57 each, in wages, to 1905 men had \$400 left. What amount had he at first?

2. Divide forty-four million, eight hundred fifty thousand, eight hundred ninety-six by eight hundred ninety-seven.

3. If I pay \$12.60 for 63 yards of merino, how many yards can I buy for \$126.00?

4. What is the sum of $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, $5\frac{5}{6}$?

5. How many barrels of meal can be bought for $\$57\frac{3}{5}$ if one is worth $\$3\frac{3}{5}$?

6. From the difference between $6\frac{8}{9}$ and $1\frac{7}{10}$ take the difference between 8 and $7\frac{8}{9}$.

7. What fraction of $\frac{5}{9}$ is $\frac{7}{18}$?

8. 56 is $\frac{7}{8}$ of what number?

9. Divide $1\frac{3}{4}$ by $7\frac{3}{8}$ and multiply the result by $\frac{8}{21}$.

10. A owned $\frac{2}{9}$ of steamship and B owned $\frac{3}{5}$. The value of the remainder was \$3200. What was the value of the vessel?

73. In exercises in which signs are used the operations denoted by $\times$ and $\div$ should be performed in the order in which these signs occur and before the operations denoted by $+$ or $-$ are performed; thus,

$$8 - 4 \div 2 + 16 - 3 \times 4 = 8 - 2 + 16 - 12 = 10.$$

74. In expressions in which "of" occurs, the numbers connected by "of" must be multiplied together before being affected by any other number; thus,

$\frac{3}{4}$ of $\frac{2}{3} \div \frac{5}{6}$ of $\frac{7}{10}$.

$= \frac{1}{2} \div \frac{7}{12} = \frac{12}{14} = \frac{6}{7}$.

This is the distinction between the use of "of" and $\times$. If instead of "of" we had in the above expression the symbol $\times$, the expression would be as follows:

$\frac{3}{4} \times \frac{2}{3} \div \frac{5}{6} \times \frac{7}{10}$. $= \frac{1}{2} \div \frac{5}{6} \times \frac{7}{10}$. $= \frac{1}{2} \times \frac{6}{5} \times \frac{7}{10} = \frac{21}{50}$.

EXERCISE XXXIV.

Simplify:

1. $6 - \frac{2}{3}$ of $\frac{5}{6} + 1\frac{1}{4}$.

2. $(\frac{2}{3} + \frac{5}{6}) \times \frac{5}{8} - \frac{7}{12}$.

3. $(\frac{2}{3} + \frac{5}{6}) \times (\frac{5}{8} - \frac{7}{12})$.

4. $\frac{2}{3} + \frac{5}{6} \times \frac{5}{8} \div \frac{7}{12}$.

5. $\frac{4}{5}$ of $3\frac{1}{8} \div \frac{4}{9}$ of 36.

6. $(8\frac{7}{11} + \frac{5}{22}) \div (8\frac{7}{11} - \frac{5}{22})$.

7. $18\frac{7}{10} - \frac{2}{3}$ of $(\frac{5}{6} - \frac{7}{18})$.

8. $1\frac{1}{12} \times \frac{21}{42} \frac{18}{5} \div \frac{10}{28} \frac{9}{9} \times \frac{25}{39} \frac{}{0}$.

9. $(1\frac{3}{7} + \frac{5}{9}) \times (1\frac{3}{7} - \frac{5}{9}) \div \frac{1\frac{3}{7}}{\frac{5}{9}}$

10. $(\frac{1}{9} + \frac{1}{10}) \div (\frac{3}{4} + \frac{5}{6})$ of $(16 - 15\frac{1}{8})$.

EXAMPLE 43. How many cents in $\$\frac{3}{7}$?

$\frac{3}{7}$ of \$1 = $\frac{3}{7}$ of 100 cents.

$= \frac{3 \times 100}{7} = \frac{300}{7} = 42\frac{6}{7}$c.

Express in dollars and cents:

$\$14\frac{3}{8}$.

EXAMPLE 44. $\$14\frac{3}{8} = \$14 + \frac{3}{8}$ of a dollar.

$= \$14 + \frac{3}{8}$ of 100 cents.

$= \$14 + 37\frac{1}{2}$ cents.

$= \$14.37\frac{1}{2}$

EXERCISE XXXV.

Express in dollars and cents:

1. $\$\frac{2}{5}$, $\$\frac{7}{10}$, $\$\frac{9}{20}$, $\$\frac{7}{11}$.
2. $\$14\frac{4}{5}$, $\$37\frac{7}{8}$, $\$8\frac{3}{10}$, $\$1\frac{5}{6}$
3. $\$17\frac{3}{4}$, $\$3\frac{9}{20}$, $\$18\frac{5}{17}$
4. $\$10\frac{11}{20}$, $\$7\frac{11}{16}$, $\$\frac{1}{100}$.
5. $\$\frac{1}{300}$, $\$6\frac{1}{50}$, $\$3\frac{11}{400}$.

EXAMPLE 45. If $2\frac{3}{4}$ yds. of tweed cost \$3.30, what will $1\frac{1}{8}$ yds. come to?

$2\frac{3}{4}$ yds. cost 330 cents

1 " costs $\frac{330}{2\frac{3}{4}}$ "

$1\frac{1}{8}$ " cost $\frac{330 \times 1\frac{1}{8}}{2\frac{3}{4}}$

$= \frac{330 \times \frac{9}{8}}{\frac{11}{4}} = 135$ cents $= \$1.35$.

EXAMPLE 46. If $1\frac{2}{3}$ bushel of oats cost $\$\frac{3}{4}$, what will $3\frac{1}{2}$ come to?

$1\frac{2}{3}$ bushel of oats cost $\$\frac{3}{4}$

1 " " costs $\frac{\frac{3}{4}}{1\frac{2}{3}}$

$3\frac{1}{2}$ " " cost $\frac{\frac{3}{4} \times 3\frac{1}{2}}{1\frac{2}{3}} = \$1\frac{2}{4}\frac{3}{0} = \$1.57\frac{1}{2}$.

EXERCISE XXXVI.

1. If $2\frac{1}{4}$ tons of hay cost \$22.50, what will $3\frac{1}{5}$ come to?
2. Paid \$9.75 for $3\frac{3}{12}$ doz. of handkerchiefs. At the same rate what would $4\frac{3}{4}$ doz. cost?

3. $3\frac{3}{7}$ yards of flannel cost $\$7\frac{7}{8}$, what is the value of $2\frac{4}{5}$ yards?

4. Paid \$1.32 for $3\frac{3}{10}$ gallons of berries. What must I pay for $10\frac{1}{2}$ gallons?

5. If $10\frac{3}{7}$ lbs. of black tea are worth $7\frac{5}{8}$ lbs. of green tea, how many lbs. of black tea are 61 lbs. of green tea worth?

6. Bought $3\frac{1}{2}$ yds. of doeskin @ \$1.40, and $3\frac{3}{8}$ yds. of pilot cloth @ \$2.40. How many yds. of flannel at $\$\frac{3}{5}$ a yard will pay the bill?

7. A man mowed $2\frac{3}{11}$ acres in $2\frac{1}{8}$ days, how many acres would he mow in $17\frac{1}{2}$ days?

8. If a man could cut $2\frac{3}{4}$ cords of wood in $3\frac{3}{4}$ days, in how many days would he cut 22 cords?

9. If $\frac{2}{3}$ of a farm was sold for \$1600, what would $\frac{1}{2}$ of the remainder be worth?

10. Paid $\$6\frac{7}{8}$ for $3\frac{1}{5}$ yds. of silk. What quantity could I buy for \$41.25?

EXERCISE XXXVII.

Miscellaneous Examples:

1. A farmer paid $\frac{1}{5}$ of his money for groceries and $\frac{3}{4}$ of the remainder for stock and had \$100 left. How much money had he at first?

2. The smaller of two numbers is $\frac{6\frac{3}{4}}{8\frac{1}{2}}$ and their difference is $3\frac{5}{6}$, what is the larger number?

3. What number multiplied by $\frac{2}{3}$ of $\frac{7}{9}$ will give one as a product?

4. A can do a piece of work in 4 days and B in 6 days. How much will A and B do in 1 day? How long will it take them to do the work if they both work at it together?

5. A can do a piece of work in 3 days, B in 4 days and C in 5. How long will it take them if they work together?

6. What is the smallest number that must be taken from 60 so that the remainder can be exactly divided by 9?

7. What is the smallest number that must be taken from 60 that the remainder may be exactly divisible by $6\frac{3}{4}$?

8. The product of 3 numbers is $3\frac{1}{4}\frac{3}{0}$. Two of the numbers are $7\frac{3}{5}$ and $8\frac{5}{1\,8}$. What is the third?

9. I bought a horse and buggy for $300. The buggy cost $\frac{2}{3}$ as much as the horse. What was the horse worth?

10. $\frac{3}{4}$ of A's money is equal to $\frac{5}{6}$ of B's, and B has $60. How much has A?

11. A's salary is $\frac{2}{3}$ of B's, and their salaries, added together, make $1000. How much is the salary of each?

12. Divide the product of $3\frac{3}{4}$ and $1\frac{4}{5}$ by their sum.

13. $\frac{7}{1\,0}$ is $\frac{5}{6}$ of what number?

14. A got $\frac{1}{6}$ of a certain sum of money and B got $\frac{1}{7}$ of the same sum. A's share was $8 more than B's. What was the sum and how much did each get?

15. What is the product of the sum and difference of $8\frac{1}{9}$ and $7\frac{3}{8}$?

16. After paying $6\frac{3}{8}$ for a coat and $2\frac{4}{5}$ for an umbrella I have $\frac{4}{5}$ of my money left. How much had I at first?

17. A man bought 7 sheep for $22 and sold 5 of them for $19. How much did he gain on each one he sold?

18. A man can do a piece of work in 10 days by working $8\frac{1}{2}$ hours a day. In how many days can he do the work if he works 10 hours a day?

EXAMINATION PAPER. No. 7.

(TIME: ONE HOUR. VALUE OF EACH QUESTION: 10).

1. Write in figures four hundred six trillion, one billion seventy-one, and write in words 640,863,000,094.

2. A merchant owned $\frac{3}{8}$ of a ship and sold $\frac{3}{4}$ of his share for $18,000. What was the whole ship worth, valued at the same rate?

3. 60 is $\frac{2}{9}$ of what number?

4. 60 is the difference between $\frac{5}{9}$ of a certain number and $\frac{2}{3}$ of it. What is the number?

5. $\frac{2}{5}$ of a number, added to $\frac{3}{8}$ of it, is equal to 62. What is the number?

6. A has twice as much money as B, and B twice as much as C, and they have altogether $14.40. How much has each?

7. Simplify $\dfrac{1\frac{1}{4}-\frac{5}{12}}{1\frac{1}{4}+\frac{5}{12}}+\frac{6}{7}$ of $\dfrac{9\times 5}{14\times 3}-\dfrac{11\frac{1}{4}}{15}$

8. How many barrels of flour, at $4.20 a bbl., can be bought for 60 bushels of potatoes, at 25 cents a bushel, 180 bushels of oats, at 40 cents a bushel, and 16 dozen of eggs, at 11 cents a dozen?

9. On $\frac{2}{5}$ of my farm I sowed oats and on $\frac{3}{4}$ of the remainder I sowed wheat. What was then left was 30 acres, on which I grew hay. How many acres were in the farm?

10. A can mow a field in 5 days, and B in 6 days. In how many days can they mow it, working together?

DECIMALS.

75. ORAL.—*(Use Blackboard).*

1. If I divide a dollar into ten equal parts what part of the dollar is one of the parts? Two of them?

2. If I divide $\frac{1}{10}$ of a dollar into ten equal parts, what fraction of the dollar is one of them? 3 of them? 35 of them?

3. If $\frac{1}{100}$ of a piece of land was subdivided into ten equal parts what part of the whole would one of them be?

4. What part of $\frac{3}{10}$ is $\frac{3}{100}$? What part of $\frac{3}{100}$ is $\frac{3}{1000}$? Of $\frac{3}{10}$ is $\frac{3}{1000}$?

$\$\frac{17}{100}=\$.17$. What takes the place of the common fraction $\frac{17}{100}$? What part of a dollar does the 1 denote? What part does the 7 denote?

5. How many hundredths of a dollar does $.17 denote?

$\$\frac{3}{10}+\$\frac{9}{100}=\$.39$.

6. In the expression $.39, what part of a dollar does the 3 denote? What part of a dollar does the 9 denote? What fraction of a dollar does the .39 denote?

76. A **Decimal Fraction** is one whose denominator is 10 or the product of 10 used as a factor two or more times, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{3}{1000}$, $\frac{7}{10000}$ are decimal fractions.

77. In writing **decimal** fractions the denominator is usually omitted, the position of the figures of the numerator to the right of a certain point, called the decimal point, indicating what the numerator is, thus:

$\frac{3}{10}$ is written .3. $\frac{5}{100}$ is written .05. $\frac{7}{1000}$ is written .007.

When the fraction is written in this manner, without writing **the denominator, the expression is simply called a decimal.**

78. The first place or order to the right of the decimal point is for *tenths;* the second for *hundredths;* the third for *thousandths;* the fourth for *ten-thousandths;* the fifth for *hundred-thousandths;* the sixth for *millionths*, etc.

.3 is read, three-tenths ($\frac{3}{10}$).

.34 is read, thirty-four hundredths ($\frac{3}{10}+\frac{4}{100}$).

.034 is read, thirty-four thousandths ($\frac{3}{100}+\frac{4}{1000}$).

.0407 is read, four hundred seven ten-thousandths ($\frac{4}{100}+\frac{7}{10000}$).

16.087 is read, sixteen *and* eighty-seven thousandths ($16+\frac{8}{100}+\frac{7}{1000}$).

406.1001 is read, four hundred six *and* one thousand one ten-thousandths ($406+\frac{1}{10}+\frac{1}{10000}$).

79 It will be seen in the system of writing decimal fractions, as in writing whole numbers, that any figure is ten times the value of the same figure written one place to the right, or $\frac{1}{10}$ its value written one place to the left.

In the whole number 666, the 6 in the tens' place is ten times the value of the 6 in the units' place and $\frac{1}{10}$ the value of that in the hundreds' place, and in the decimal .666, the 6 in the hundredths' place is ten times the value of the 6 in the thousandths' place and $\frac{1}{10}$ the value of the 6 in the tenths' place. In the expression 4444.4444 the value of the figure 4 in any place is ten times the value of the 4 next on the right and $\frac{1}{10}$ the value of the 4 next on the left.

In reading whole numbers the word "and" is not used at all. In reading mixed numbers the word "and" is only used when we come to the decimal point. 200.063 is read, two hundred *and* sixty-three thousandths.

.263 is read, two hundred sixty-three thousandths. To avoid ambiguity it is sometimes necessary to pause before the denominator

of the decimal; thus .400 is four hundred (pause) thousandths, and .00004 is four (pause) hundred-thousandths.

When decimals are much used it is convenient to simply pronounce the word "decimal," or "point," at the decimal point and then name the digits in order. Thus, .864 is frequently read decimal (or point) eight, six, four, and 406.087 is read, four hundred six decimal (or point) nought, eight, seven.

Pupils should not use this method until thoroughly familiar with the notation and numeration of decimals.

EXERCISE XXXVIII.

Read the following:

1. .5; .05; .005; .0005.
2. .0008; .00008; .000008.
3. .0000009; .00016.
4. .37; .037; .00037.
5. .0368; .00368; .000368.
6. .0406; .001001; .3600.
7. .2036009; .08060011.
8. .004104; .0807016.
9. .2027016; .3060018.
10. .400161; .000400161.

80. In the above exercise we read the number as if it were a whole number and then apply the denomination indicated by the position of the right hand figure of the numerator.

81. Mixed numbers may be read in two ways; thus, 4.3 is 4 *and* three tenths or 43 tenths; 64.08 is sixty-four and eight hundredths, or six thousand four hundred eight hundredths.

EXERCISE XXXIX.

Read the following in both ways:

1. 6.4; 6.44; 6.04.
2. 63.8; 63.08; 63.008.
3. 846.86; 846.0086; 3680.0046.
4. 87.0089; 876.0001001.
5. 9663074.087; 1060.00870.

EXAMPLE 47. Write thirty-four thousandths ($\frac{34}{1000}$) as a decimal. Since the denomination "thousandths" is in the third place after the decimal point there must be three digits after the decimal point. As thirty-four has only two digits we must put a nought after the decimal

point to make up the required number of digits. The decimal is written .034.

82. Write the numerator of the decimal and if necessary put noughts to the left of it to make up the number of places required by the denomination, and place the decimal point to the left of the tenths.

EXERCISE XL.

Write as decimals:

1. $\frac{7}{10}$, $\frac{3}{10}$, $\frac{9}{10}$, $\frac{5}{10}$.

2. $\frac{3}{100}$, $\frac{17}{100}$, $\frac{19}{100}$, $\frac{75}{100}$.

3. $\frac{44}{1000}$, $\frac{4}{1000}$, $\frac{183}{1000}$.

4. $\frac{4}{10000}$, $\frac{16}{10000}$, $\frac{196}{1000000}$.

5. $6\frac{7}{10}$, $186\frac{19}{100}$, $26\frac{14}{10000}$.

6. Seven hundredths; forty-four thousandths.

7. Four hundred six ten-thousandths.

8. Five hundred thousandths; five hundred-thousandths.

9. Two hundred millionths; two hundred-millionths.

10. Four hundred and nineteen hundred-thousandths.

11. Six thousand four and one ten-millionth.

12. One million three thousand one and seventeen billionths.

EXAMPLE 48. Reduce .64 to a common fraction.

$$.64 = \frac{64}{100} = \frac{16}{25}.$$

EXAMPLE 49. Reduce .0064 to a common fraction.

$$.0064 = \frac{64}{10000} = \frac{8}{1250}.$$

EXERCISE XLI.

Write as common fractions:

1. .8.
2. .12.
3. .146.
4. .08.
5. .016.
6. .0084.
7. .02074.
8. .00083.
9. .40694.
10. .30807.
11. .00367.
12. .000007.
13. 6.84.
14. 17.096.
15. 143.008.
16. 176.304.
17. 6.0007.
18. 1.00016.

$.7 = \frac{7}{10}$. $.70 = \frac{70}{100} = \frac{7}{10}$. $.700 = \frac{700}{1000} = \frac{7}{10}$.

It is evident from the above that .7, .70 and .700 are all equal and that one or more noughts annexed to the right of a decimal do not change its value, thus:

$.9 = .90 = .900 = .9000$.

ADDITION OF DECIMALS.

EXAMPLE 50. Find the sum of .608, .0467 and .64.

.608	We place the addends so that the tenths will be under the tenths and the hundredths under the hundredths, and so on with the other figures. We add as in whole numbers putting the decimal point to the left of the tenths. Any number carried from the tenths is put in the units place.
.0467	
.64	
1.2947	

EXAMPLE 51. Find the sum of 6.38, 7.046, .0864, 64.089.

6.38
7.046
.0864
64.089
77.6014

EXERCISE XLII.

Find the sum of:

1. .06 + .08 + .019.
2. .84 + .648 + .7364 + .96.
3. .734 + .869 + .6374.
4. 6.34 + 7.375 + 4.645.
5. 86.087 + 96.0845 + .067.
6. 119.47 + .063 + 17.384.
7. 63.0004 + 17.004 + .04 + .4.
8. .0064 + .064 + .64 + 6.4 + 64.
9. 63 + .08 + 1.8 + 17 + 63.17.
10. 63.17 + 8.19 + 64 + 1.7 + .0008.
11. 75 + 7.5 + .75 + .075 + .0075.
12. 8638 + 863.8 + 86.38 + 8.638 + .8638.

13. A ship sailed 10.7 miles on the first day out, 11.8 on the second, 11.9 on the third, 4.37 on the fourth, and 12.8 on the fifth. How many miles were sailed in all?

14. A had \$17.305, B \$17.855, C \$14.44, and D \$16.835. What had they all together?

15. Five fields contain respectively 6.38, 4.94, 7.35, 5.9 and 8.105 acres. How many acres were in the lot?

SUBTRACTION OF DECIMALS.

EXAMPLE 52. From 46.84 take 16.0465.

46.8400		46.84
16.0465	or	16.0465
30.7935		30.7935

Place the subtrahend under the minuend, with the tenths under the tenths, the hundredths under the hundredths, &c. Subtract as in whole numbers taking care to place the decimal point immediately to the left of the tenths. When there are more digits to the right of the decimal point in the subtrahend than there are to the right of the decimal point in the minuend, naughts may be added to make the number of digits equal.

EXERCISE XLIII.

From	**1.** 17.9	**2.** 14.	**3.** 5.846
take	3.9673	3.076	.987
	4. 17.846	**5.** 1.	**6.** 19.846
	.9	.0009	14.

Find the value of:

7. 36.84 – 17.85.
8. 36.8 – 36.08.
9. 17 – .004.
10. 36.04 – 1.9999.
11. 1000 – .999.
12. 49.8 – 1.998.
13. 8.644 – .234.
14. 3 – 2.904.
15. 136 – .099.
16. 1000 – .0009.
17. 17.384 – 16.384001.
18. 163.00847 – 63.00848.
19. \$463.084 – \$.75.
20. \$63.10 – \$10.09.

21. From three hundred and seven ten-thousandths take three hundred and seven hundred-thousandths.

22. From one hundred four and sixteen millionths take one thousand, eight hundred five ten-thousandths.

23. The beer gallon contains 282 cubic inches and the imperial gallon 277.274 cubic inches. How many more cubic inches in the beer gallon than in the imperial?

24. The wine gallon contains 231 cubic inches. What is the difference between it and the imperial gallon?

MULTIPLICATION OF DECIMALS.

83. Since $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = .06$
Therefore $.3 \times .2 = .06$.
And since $\frac{3}{100} \times \frac{4}{100} = \frac{12}{10000} = .0012$.
Therefore $.03 \times .04 = .0012$.
Also since $12\frac{7}{10} \times 5\frac{9}{100} = \frac{6461}{100} = 64.643$.
Therefore $12.7 \times 5.09 = 64.643$.

84. Notice in the three examples above that when decimals are multiplied the number of decimal places in the product is the sum of the decimal places in the multiplicand and multiplier. Try other decimals.

	Example 53.	Example 54.	Example 55.
	.83	.304	6.32
Multiply by	.24	.015	4.06
	332	1520	3792
	166	304	2528
	.1992	.004560	25.6592

85. Multiply as in whole numbers and from the right hand of the product point off as many figures for decimals as there are decimal places in the multiplicand and multiplier. If the product does not contain as many figures as there are decimal places in the multiplicand and multiplier prefix noughts to make up the number of places required.

EXERCISE XLIV.

Find the product of:

1. $.7 \times .8$.
2. $.16 \times .84$.
3. $.08 \times .72$.
4. $.06 \times .03$.
5. $.04 \times 4.3$.
6. 6.04×4.07.
7. $.0432 \times 5.4$.
8. $.0009 \times .008$.
9. $450 \times .006$.
10. $.018 \times 4680$.

11. $.675 \times 6.75$.
12. 214.76×89.104.
13. $1064 \times .0008$.
14. $436 \times .086$.
15. $3.45 \times .000345$.
16. $6.8 \times .1$.
17. $1000 \times .005$.
18. $76 \times .00001$.
19. 4.38×16.74.
20. 83.61×100.06.
21. 4.943×55.01.
22. $36.001 \times .008$.
23. 4.2008×1.25.
24. 87.03×8.412.
25. 36.84×1.008.
26. 87.06×3.0016.

86. To multiply by 1 followed by one or more noughts:

$8.463 \times 10 = 84.63$. $8.463 \times 1000 = 8463$.

$8.463 \times 100 = 846.3$. $8.463 \times 10000 = 84630$.

From the above examples it will be seen that in each case the decimal point of the multiplicand has been moved as many places forward in the product as there are noughts in the multiplier.

87. To multiply by 10, 100, 1000, etc., move the decimal point as many places to the right in the multiplicand as there are noughts in the multiplier and the result will be the product.

EXERCISE XLV.

1. How much must be paid for 36.5 bushels of wheat @ $.75 per bushel?

2. If a rolling mill makes 106.8 tons of iron in a day, how much will it make in 124.25 days?

3. A meter measures 39.37 inches. How many inches in 84.75 meters?

4. The circumference of any circle is equal to its diameter multiplied by 3.1416. What is the circumference of a circle whose diameter is 3.08 yards?

5. What is the product of the sum and difference of .75 and .5?.

6. Multiply two hundred fourteen and sixty-two millionths by one and three thousand seven hundred-thousandths.

7. A pint of water weighs 1.25 lbs. avoirdupois, what is the weight of 6.75 pints?

8. A gram weighs 15.432 grains. How many grains in 17.384 grams?

9. The standard bushel of Canada contains 8 standard gallons, of 277.274 cubic inches each. How many cubic inches in it?

10. A man earns $10.875 a week and spends $1.375 every day of the seven days of the week. How much does he save in 25 weeks?

EXAMINATION PAPER. No. 8.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10).

1. Multiply the product of forty-six and four hundredths by their difference?

2. A poulterer sold 30 geese at $.40, and lost $3 on the lot. What would he have gained had he sold them for $.55 apiece?

3. What is the G. C. D. of 362,754 and 987,399?

4. If 49 yards of tweed cost $52.92, what will 343 yards cost?

5. Simplify $(6 - .0095) \times .0036$.

6. What part of $\frac{8}{11}$ is $\frac{8}{33}$?

7. $\frac{5}{12}$ is $\frac{1}{3}$ of what number?

8. A and B have $141.00 between them, but B's money is $20.20 more than A's. What is A's money?

9. A boy spent 40 cents more than $\frac{2}{3}$ of his money and had 16 cents left. How much money had he at first?

10. A's salary is $\frac{11}{12}$ of B's and $\frac{3}{5}$ of B's is equal to $360. What is A's salary?

DIVISION OF DECIMALS.

88. ORAL (*Use Blackboard*).

1. What number, multiplied by 6, will give 3 tenths? .05.
2. What number, multiplied by 6, will give 3 hundredths? .005.
3. $.3 \div 6 = ?$ $.03 \div 6 = ?$
4. $.84 \div 12 = ?$ $.084 \div 12 = ?$
5. 24 | .072 | ?
6. 36 | .0072 | ?
7. 36 | .00432 | ?

Let the pupil observe that when the divisor is a whole number, the quotient is of the same denomination as the dividend.

DIVISION OF DECIMALS.

Example 56. Divide 858 by 24.

24|858|$35\frac{3}{4}$
72
138
120
18
$\frac{18}{24} = \frac{3}{4}$

or

24|858|35.75
72
138
120
180 tenths
168
120 hundredths
120

We proceed as above until we come to the remainder 18, which is 18 units. Since 18 units = 180 tenths we proceed to find what number multiplied by 24 will give 180 tenths. 7 tenths multiplied by 24 gives 168 tenths. We put 7 tenths in the quotient and subtract. The remainder is 12 tenths, which is equal to 120 hundredths. 5 hundredths multiplied by 24 gives 120 hundredths. We therefore put 5 hundredths in the quotient. Observe that the denomination of any figure in the quotient is the same as that of the partial dividend that produces it.

Example 57. Divide .0858 by 24.

24|.0858|.003575
72
138 ten-thousandths
120
180 hundred-thousandths
168
120 millionths
120

Here we ask what number multiplied by 24 will give 85 thousandths. 3 thousandths comes nearest, which multiplied by 24 gives 72 thousandths. We therefore put 3 thousandths (which is of course written .003) in the quotient and continue as before.

Example 58. Divide 5 by 625.

625|5.000|.008
5 000

To find what number multiplied by 625 will give 5, we must first reduce 5 to thousandths. Then 8 thousandths multiplied by 625 gives 5. We put .008 in the quotient.

To divide a whole number or a number having decimals by a whole number:

89. Divide as in division of whole numbers, taking care that each significant figure of the quotient is of the same denomination as the denomination of the dividend or partial dividend that produces it.

EXERCISE XLVI.

Divide:

1. 57.6 by 18.
2. 172.8 by 36.
3. 15.552 by 72.
4. 165.78 by 54.
5. 48.591 by 90.
6. 9.6188 by 346.
7. .005 by 100.
8. .03456 by 144.
9. .001 by 25.
10. 27 by 450.
11. .15 by 1875.
12. .014274 by 61.
13. .029295 by 465.
14. .1 by 10000.
15. 20.7861 by 579.

To divide when the divisor is a decimal or a whole number and a decimal.

EXAMPLE 59. Divide 17.28 by 14.4:

$$\frac{17.28}{14.4} = \frac{17.28 \times 10}{14.4 \times 10} = \frac{172.8}{144} = 1.2.$$

Or

$$\begin{array}{r|l} 14.4 & 17.28 \\ \hline 144 & 172.8 \end{array} \Big| 1.2$$

$$\begin{array}{r} 144 \\ \hline 288 \\ 288 \\ \hline \end{array}$$

Here we multiply the divisor and dividend by 10 and make the divisor a whole number.

EXAMPLE 60. Divide .05 by .00025.

$$\begin{array}{r|l} .00025 & .05 \\ \hline 25 & 05000 \end{array} \Big| 200$$

$$\begin{array}{r} 50 \\ \hline 00 \end{array}$$

In this case we multiply divisor and dividend by 100000 and make the divisor a whole number.

90. When the divisor is a decimal make it a whole number by removing the decimal point a sufficient number of places to the right.

Move the decimal point of the dividend the same number of places to the right.

EXAMPLE 61. Divide 6 by .0125.

$$\begin{array}{r|l} .0125 & 6.0000 \\ \hline 125 & 60000 \end{array} \Big| 480$$

$$\begin{array}{r} 500 \\ \hline 1000 \\ 1000 \\ \hline 0 \end{array}$$

EXERCISE XLVII.

Divide:

1. 5.526 by 1.8.
2. 16.192 by .32.
3. 10.24 by .0064.
4. 4.126 by .64.
5. .93 by .075.
6. .0012 by 1.6.
7. .0774 by 4.8.
8. 14.41728 by 72.
9. 1 by .0001.
10. .02921 by .23.
11. .03096 by .000086
12. .24294591 by 36.9.
13. .131053 by .065.
14. 61 by .0007625.
15. 19.0026 by 16.2.
16. 830.676 by .0231.
17. .039034 by 13.46.
18. 5.0955 by .0395.
19. 20.7861 by 5.79.
20. 6522.0834 by .00427.

EXAMPLE 62. Divide 47 by 32.

$$
\begin{array}{r|l|l}
32 & 47.000 & 1.468+ \\
 & 32 & \\
 & \overline{150} & \\
 & 128 & \\
 & \overline{220} & \\
 & 192 & \\
 & \overline{280} & \\
 & 256 & \\
 & \overline{24} &
\end{array}
$$

At this stage of the operation we have the answer correct to three decimal places (1.468) and we might write it 1.468+ and cease. If we wish to carry it further we go on as before annexing a nought to the right of the dividend and to the remainder and then putting the proper figure in the quotient.

EXERCISE XLVIII.

Find the quotient as far as the sixth decimal place:

1. $8.750 \div 2.52$.
2. $.012345 \div 1.001$.
3. $80.1 \div .0624$.
4. $12000 \div 1203.06$.
5. $.015 \div 162.5$.
6. $.700 \div .0042$.
7. $3.413682 \div 49.1$.
8. $411.0125 \div 1.023$.
9. $456.789 \div 12.3$.
10. $456.8 \div 3060.125$.
11. $90 \div .00001263$.
12. $8888.8 \div 73000$.

To divide a decimal by 1, followed by one or more noughts:

$686.034 \div 10 = 68.6034$. $686.034 \div 1000 = .686034$

$686.034 \div 100 = 6.86034$. $686.034 \div 10000 = .0686034$

91. To divide a decimal by 1, followed by one or more noughts, we remove the decimal point in the dividend as many places to the left as there are noughts in the divisor.

92. In *any* exercise in division of decimals, if noughts be added so as to give the same number of decimal places in both divisor and dividend, the divisor and dividend then stand to each other as whole numbers. This is because they then have the same denominator. Then, when an *additional* nought is added to the dividend, the decimal point is placed in the quotient. This is the only method used by some successful teachers.

To reduce a vulgar fraction to a decimal:

EXAMPLE 63.—Reduce $\frac{3}{4}$ to a decimal fraction.

$$4\overline{)3.00}\quad .75$$

$\frac{3}{4}$ is equal to $\frac{1}{4}$ of 3, or 3 divided by 4.

To reduce a vulgar fraction to a decimal, divide the numerator by the denominator.

EXERCISE XLIX.

Reduce to decimal fractions:

1. $\frac{1}{4}$.
2. $\frac{1}{2}$.
3. $\frac{3}{5}$.
4. $\frac{7}{8}$.
5. $\frac{9}{10}$.
6. $\frac{15}{10}$.
7. $\frac{1}{25}$.
8. $\frac{17}{32}$.
9. $\frac{3}{250}$.
10. $\frac{1}{320}$.
11. $\frac{17}{800}$.
12. $\frac{19}{32}$.
13. $\frac{23}{160}$.
14. $64\frac{5}{8}$.
15. $.08\frac{4}{5}$.
16. $17.08\frac{3}{4}$.
17. $17\frac{1}{8}$.
18. $.17\frac{1}{8}$.
19. $5.78\frac{3}{16}$.
20. $.3\frac{11}{125}$.
21. $4.0\frac{7}{8}$.
22. $\frac{1}{160}$.
23. $\frac{17}{3125}$.
24. $\frac{11}{125}$.

EXAMINATION PAPER. NO. 9.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10).

1. Write out six hundred four million one thousand forty-nine and sixty-three trillionths.

2. A and B rented a pasture for the summer for $15.00. A pastured 20 cows in it and B 30. How much ought each to pay?

3. If $\frac{4}{5}$ of a barrel of flour is worth $\frac{5}{8}$ of a ton of coal, how many barrels can I get for $6\frac{1}{4}$ tons of coal.

4. Divide the average of $3\frac{3}{8}$, $5\frac{7}{12}$ and $7\frac{9}{16}$ by their sum.

5. Divide 6.8 by .0625.

6. A's money is equal to $\frac{2}{3}$ of B's. They have together $60. What has each?

7. A has half as much money as B and B has half as much as C. They have in all \$420. How much has each?

8. 40 is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of what number?

9. If an ounce of gold is worth \$19.45, how many ounces will buy 38.9 acres of land at \$16.27 an acre?

10. A man sold $\frac{3}{8}$ of his land for \$20 an acre and got \$3000. He sold $\frac{1}{2}$ of the remainder at \$15 an acre and the rest at \$10 an acre. How much did he get for his farm?

EXAMINATION PAPER. NO 10.

(TIME: ONE HOUR. VALUE OF EACH QUESTION, 10).

1. Multiply six hundred and four hundred-thousandths by six hundred four hundred-thousandths.

2. How many revolutions will a wheel whose circumference is 6.9 feet make in going over a distance of 1192.32 feet?

3. Find the product of the sum and difference of .5 and .05 and write it as a vulgar fraction.

4. A Canadian gallon contains 277.274 cubic inches and a United States gallon contains 231 cubic inches. What is the difference in cubic inches between 5 gallons of Canadian measure and 6 gallons of United States measure?

5. Simplify $14 \div \frac{2}{3}$ of $(\frac{7}{8} - \frac{5}{12}) \times 22$.

6. If 6.84 bushels of wheat are worth 9.6 bushels of oats, how many bushels of oats can be had for 205.2 bushels of wheat?

7. A owned $\frac{5}{9}$ of a ship and B owned $\frac{3}{7}$ of it. The value of A's share was \$8000 more than B's. What was the value of the ship?

8. What is $\frac{4}{5}$ of the number of which 50 is $\frac{5}{7}$?

9. A man can do a piece of work in 30 days. What part of it can he do in $16\frac{1}{2}$ days?

10. A can do a piece of work in 25 days and B in 30 days. How much more than B can A do in 1 day?

EXERCISE L.

Miscellaneous Examples: (*Use brackets when possible*).

1. Simplify $(84.03 + 17.8) \times (84.03 - 17.8)$.

2. What is the product of the sum and difference of .005 and .5?

3. Simplify $(10+7.08)\div(10-7.08)$.

4. Divide the sum of 9 and .007 by their difference.

5. Simplify $(\frac{3}{8}+.025-.007)\times\frac{4}{5}$.

6. Divide five hundred twelve ten-thousandths by six and four tenths.

7. Change .00975 to a common fraction.

8. If a man travel 27.45 miles a day, how far will he travel in $6\frac{4}{5}$ days?

9. The wheel of a bicycle is 9.25 feet in circumference. How often will it turn in going 37 miles (a mile is 5280 feet)?

10. A boy who was driving a horse for a farmer was to receive $\frac{1}{8}$ of a bushel of oats for every load he took. By mistake he got .124 of a bushel. How much did he gain or lose in 100 loads?

11. Simplify $\dfrac{(8.308+5.0308)\times(8.308-5.0308)}{8-7\frac{4}{5}}$

12. How many suits can be made from 195.75 yards if 7.25 yards make one suit?

13. A man spent .375 of his ready money for a horse and .25 of it for farm implements and had $120 left. How much money had he at first?

14. What difference will it make if the 3 and 5 exchange places in the expression .03456?

15. Simplify $\dfrac{\$36.50\times 8\times 27}{73000}$

16. Simplify $\dfrac{\$64.80\times .08\times\frac{3}{4}}{.016}$

17. A's money is $\frac{2}{3}$ of B's and B's is $\frac{2}{3}$ of C's. C's money is $60. What is A's?

18. A paid $16 for $\frac{2}{5}$ of an acre of land and B paid $40 for $\frac{5}{7}$ of an acre. How much more than A did B pay per acre?

19. Simplify $\dfrac{\$1.40\times 10\times 29}{73000}$

20. A raises $\frac{5}{9}$ the number of bushels of oats that B raises. $\frac{7}{12}$ of what B raises is 840 bushels. How many bushels does A raise?

21. I multiplied a certain number by 3 and added $6\frac{3}{7}$ to the product. The result was $16\frac{3}{7}$. What was the number?

22. A man paid $\$206\frac{1}{4}$ for a horse and harness. The horse cost 9 times as much as the harness. What was the harness worth?

23. A got $\frac{1}{6}$ of a certain sum of money; B got $\frac{2}{5}$ of it and C got $\frac{1}{3}$ of it. Altogether they received $270. What was the sum?

24. $\frac{5}{6}$ of A's money is equal to $\frac{2}{3}$ of B's. How much of A's is all B's. How much of B's is all A's?

DENOMINATE NUMBERS.

METRIC SYSTEM.

93. The metric system of weights and measures is used by nearly all the civilized nations of the world, except Great Britain, Canada and the United States. The system has, however, been legalized in these countries, and is destined before long to come into general use.

94. When we wish to measure a quantity, such as length, surface, volume, weight, value, heat, etc., we do so by finding how many times it contains some *known quantity* of the same kind. This known quantity is called the *unit of measure*.

95. In the metric system, the unit of measure for lengths is the **Meter**, and from it the units of measure for surface, volume, capacity and weight are derived. The length of the *meter* is 39.3704 + inches.

The length of the *meter* was intended to be one ten-millionth of the distance from the equator to the pole; but, owing to a slight

error on the part of the French scientists who made the measurements, 10,001,887 standard meters are contained in the distance which was intended to contain 10,000,000.

96. The *unit of measure* for surfaces is the *square meter*—a square, each side of which is 1 meter long.

97. The *unit of measure* for volume is the *cubic meter*—a cube, each of whose six plane surfaces is 1 square meter.

98. The *unit of measure* for capacity is the *liter* (pronounced leeter)—a cube, each edge of which is one-tenth of a meter in length.

99. The *unit of measure* for weight is the *gram*—the weight, at its greatest density, of a body of water equal in volume to a cube, each edge of which is the hundredth part of a meter.

100. From these *units of measure* all other denominations are formed by multiplying or dividing by 10, 100 or 1000.

Deci means	$\frac{1}{10}$.	Deka means	10.
Centi "	$\frac{1}{100}$.	Hekto "	100.
Milli "	$\frac{1}{1000}$.	Kilo "	1000.
		Myria "	10000.

Thus, *decimeter* means $\frac{1}{10}$ of a meter; *dekameter* means 10 meters; *centimeter*, $\frac{1}{100}$ of a meter; *hektometer*, 100 meters; *millimeter*, $\frac{1}{1000}$ of a meter; *kilometer*, 1000 meters.

MEASURES OF LENGTH.

101.

10 Millimeters (mm)	= 1 *Centimeter* (cm)	= .01 of a meter.
10 Centimeters	= 1 Decimeter (dm)	= .1 of a meter.
10 Decimeters	= 1 **Meter** (m)	= 39.37 + inches.
10 Meters	= 1 Dekameter (Dm)	= 10 meters.
10 Dekameters	= 1 Hektometer (Hm)	= 100 "
10 Hektometers	= 1 *Kilometer* (Km)	= 1000 "
10 Kilometers	= 1 Myriameter (Mm)	= 10000 "

102. The **meter**, *centimeter*, *millimeter* and *kilometer* are the measures in most common use.

103. Any length given in one denomination may be expressed in any other denomination simply by moving the decimal point to the right or left. Thus: 8295034^{mm} is 829503.4^{cm}, or 82950.34^{dm}, or 8295.034^{m}, or 829.5034^{Dm}, or 82.95034^{Hm}, or 8.295034^{Km}. It can also be written $8^{Km}\ 2^{Hm}\ 9^{Dm}\ 5^{m}\ 3^{cm}\ 4^{mm}$

The cut shows exactly $\frac{1}{10}$ of a meter. Let it be used by the class to prepare a meter-stick, with proper sub-divisions, for practical measurements; also blocks to represent the cubic centimeter and liter (cubic decimeter). This cubic centimeter block shows the bulk of water which, at 4 degrees Centigrade (4° C.) weighs 1 gram.

If a vessel, perfectly full of cold water, be placed in a tin pan or other empty vessel, and a cubic block, each edge of which is $\frac{1}{10}$ of a meter long, be sunk in the water, one liter of water will run over into the empty vessel. This liter of water weighs 1000 grams.

$\frac{1}{10}$ of a meter = 1 decimeter = 10 centimeters = 100 millimeters.

104. ORAL.

1. What is the length and breadth of your book in centimeters? Its thickness in millimeters?

2. What is the length of your desk in meters and centimeters? Also the height and width of the door?

3. Find the length, breadth and height of the school room in meters and centimeters? Also your own height?

4. How many meters in a kilometer? How many centimeters?

105. Before adding or subtracting quantities we must first express them in the same denomination or *unit of measure.* Thus:

EXAMPLE 64. Add 25^{dm}, 76^{m}, 86^{mm}, 18^{Km} and 195^{cm}

Meters.

2.5
76
.086
18000
1.95

18080.536m.

EXERCISE LI.

Find the value of the following expressions in meters or in dollars:

1. $24^{m} + 935^{cm} + 25^{dm} + 2^{Km} + 679^{mm}$

2. $712^{m} + 712^{dm} + 712^{cm} + 712^{mm} + 712^{Hm}$

3. $19.41^{Km} + 194.1^{Hm} + 1941^{m} + 7.235^{m} + 72.35^{dm} + 723.5^{cm} + 7235^{mm}$

4. $16.37^{m} + 9.276^{Dm} + 500^{m} + 6.388^{m} + 745^{dm} + 2^{dm}$

5. \$17.05 + \$200 + 86 cents + \$3.145 + cents 256 + \$.765 + \$19.

6. (\$25.60 + \$9.115 − \$18.75) × 9.

7. $(24.3^{m} + 95.65^{m} + 130^{m}) \times 12$.

8. $(4^{Km} + 734^{m} + 8.74^{m} + 86^{cm}) \times 24$.

9. $8067^{m} - 286.45^{m}$

10. $3.17^{Km} - 486.254^{m}$

11. $9.93^{Km} - (5684.2^{m} + 766.5^{m} + 1925.88^{m})$

12. $192^{Km} + 84.377^{Km} + 9.8864^{Km} - 277^{Km}$

13. $(1.3084^{Km} + 13.084^{Hm} + 130.84^{Dm} + 1308.4^{m} + 13084^{dm}) \times 34$.

14. $117.11^{m} \div 25$.

15. $(72.4^{m} \times 5 + 103.5^{m} \times 6) \div 5$.

16. A lady bought 7.4^{m} of lustre, 6.1^{m} of serge, 13.1^{m} of cotton and 75^{cm} of silk. How many meters did she buy?

17. A dining table which was 6.75^{m} long had 3 boards each 36^{cm} wide inserted into it. What was the length of the table then?

18. A post which is 4.125^{m} long is 1.16^{m} under ground. What is the length of the part above ground?

19. In a certain garden fence there are 12 panels, each panel has 5 planks and each plank is 5.315^{m} long. What is the total length of plank?

20. If the length of the school-room is 8^{m} and the breadth 5.435^{m}, what distance would you walk in going around the room 17 times?

21. At \$1.25 per meter, what is the cost of 6.25^{m} Canadian tweed?

22. What is the value of 13.40^{m} of cloth when 17.60^{m} cost \$25.52?

23. If the cost of a railroad ticket to travel 145^{Km} is \$3.19, what must be paid for a ticket good for 613^{Km}?

24. A boy who lives 930^{m} from the post office goes to mail a letter and returns. How many steps of an average length of 62^{cm} does he require to take?

25. From a piece of cloth containing 28.60^{m} a tailor

Inches.
Centimeters.

cut off 3 pieces: the first of 2.72^{m}, the second 95^{cm} and the third 3.25^{m}. How much cloth is left?

26. How many suits each requiring 6.65^{m} can be made from a web of broad-cloth containing 36^{m}?

27. How often must a wheel 3.2^{m} in circumference revolve in going 3.84^{Km}?

28. The distance between two railroad stations is 63.54^{Km}, and the distance between two telegraph poles is 90^{m}. How many telegraph poles will be needed between the two stations?

SURFACE MEASURES.

106. The principal *unit of surface* is a square each side of which is one meter long, called a *square meter.*

Ten pieces of cardboard, each one decimeter square, if placed after each other in a row would make a strip 1 meter long and 1 decimeter wide. Ten such strips side by side would give a square 10^{dm} or 1^{m} in length and 10^{dm} or 1^{m} in breadth, or 1 square meter. Now as each strip contains 10 square decimeters the 10 strips must contain 10×10 or 100 square decimeters. Hence 1 square meter = 100 square decimeters. In surface measure then it will be understood that 100 of any denomination will be required to make 1 of the next higher.

107.

		1 sq. millimeter	= .000001	of a	sq.	meter.
100 sq millimeters (qmm.)	= 1	" *centimeter*	= .0001	"	"	"
100 " centimeters (qcm.)	= 1	" decimeter	= .01	"	"	"
100 " decimeters (qdm.)	= 1	" **meter**	= 1.196	square	yards.	
100 " meters (qm.)	= 1	" dekameter	= 100	square	meters.	
100 " dekameters (qDm.)	= 1	" hektometer	= 10000	"	"	
100 " hektometers (qHm.)	= 1	" *kilometer* (qKm.)	= 1000000	"	"	

A square centimeter showing square millimeters.

108. In measuring land the square dekameter,—a square each side of which is 10^{m} long, is called an **ar** (pronounced *air*), and the square hektometer each side of which

is 100^{m} long is called a **hektar** (*hektair*). The square meter is called a **centar** (*centair*).

100 centars$^{(ca)}$ = 1 ar$^{(a)}$
100 ars = 1 hektar$^{(Ha)}$

109. A flat surface, enclosed by four straight lines, and having four square corners or right angles, is called a **rectangle**. If the sides are all equal it is called a **square**.

110. ORAL.

(The teacher should see that the pupil actually *does* the work suggested).

1. Draw on the board a square meter. Divide each vertical side by points into 10 equal parts, or decimeters. Join the two points nearest the lower side of the meter, then the next two and so on. How many decimeters long and wide is the first strip thus formed? 10^{dm} long, 1^{dm} wide. Divide each long side of this strip into 10 equal parts or into decimeters. Join the points directly opposite each other. How many squares, or square decimeters, are there in this strip? How many such strips in the meter? How many square decimeters in a square surface each side of which is 10^{dm} long? In a square each side of which is 5^{dm} long? 9^{dm}? 4^{dm}? 7^{dm}? 3^{dm}?

2. Draw on the board a rectangular figure 7^{dm} long and 3^{dm} wide. Divide each side by points into parts each one decimeter long. Join the points directly opposite each other. How many square decimeters are there? What must you do with the numbers representing the length and breadth of your figure to get 21?

3. In a hall 20^{m} long and 10^{m} wide how many square mats, each 1 meter square, would you require to place one after the other to make a strip the length of the hall? To make two strips? 3? 4? 5? 6? 7? 8? 9? To cover the whole floor? How many square meters of carpet would cover that floor?

4. How many square meters in the ceiling of the above-mentioned hall? In one side wall, if it is 5^{m} high? In one end wall? In the four walls? In the whole interior surface of the hall?

5. How many square meters in a rectangular garden-plot 12^{m} long and 10^{m} wide?

6. What is the breadth of a rectangular flower-bed 12 meters long, containing 120 square meters? When the breadth is 10^{m} what is the length? The length of another is 9^{m} and area 63^{qm}. What is the breadth?

EXERCISE LII.

1. A room 4^{m} long and 3^{m} wide contains how many square decimeters?

2. How many square meters in a room 5.15^{m} long 4.4^{m} wide?

3. What would it cost to put oil-cloth on the floor of a room 5^{m} long and 4^{m} wide at \$1.50 per square meter?

4. What would it cost to silver a mirror 1.4^{m} long and 1^{m} wide at $\frac{1}{10}$ of a cent per square centimeter?

5. How many square meters in a web of carpet 80^{m} long and $.85^{m}$ wide?

6. A room which is 6.11^{m} long and 4.5^{m} wide contains how many square meters?

7. A room which is 6.11^{m} long contains 27.495^{qm}. What is its breadth?

8. A room which contains 19.712^{qm} has a breadth of 3.85^{m}. What is its length?

9. In 86.4^{qm} how many square centimeters?

10. In 287659^{qmm} how many square meters? Square kilometers?

11. Make a rectangular diagram 5^{cm} long and 3^{cm} wide. Divide it up so as to show that the product of its length and breadth equals the number of square centimeters it contains.

12. A walk 120^{m} long contains 218.4^{qm}. What is its breadth?

13. In a field 500^{m} long and 400^{m} wide contains how many centars? How many ars? Hektars?

14. From a farm of 80^{Ha} a piece containing $4^{Ha}\ 56^{a}$ is sold. What is the value of the remainder at \$30 per hektar?

15. A field containing 4^{Ha}, which cost \$950, was divided into lots 80^{m} long and 25^{m} wide. The lots were sold at \$65 each. What was the gain?

MEASURES OF VOLUME.

111. ORAL.

How many blocks, each 1 decimeter long, 1 decimeter wide and 1 decimeter thick, would it be necessary to place one after another to make a row 1 meter long? 10. How many of these rows would have to be placed side by side to make a width of 1 meter? 10. How many of the blocks (cubic decimeters) would be in the 10 rows?

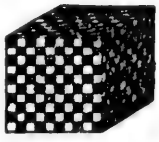
100. What would be the length of the space covered? 1 meter. The breadth? 1 meter. How many cubic decimeter blocks would make one layer over a square meter? 100. How many would make two layers? 200. Three layers? Four? Five? Six? Seven? Eight? Nine? Ten? 1000. What is the height of one layer? 1 decimeter. Of two? Three? Five? Ten? 10 decimeters or 1 meter. What is the length of the pile of blocks? 1 meter. The breadth? 1 meter. The height? 1 meter. What is the form of the pile? A cube. What would you call the pile? A cubic meter. How many cubic decimeters in a pile which is 10^{dm} long, 10^{dm} wide and 10^{dm} high? $10 \times 10 \times 10$ or 1000 cubic decimeters.

Hence in measures of volume 1000 of any denomination is required to make 1 of the next higher.

112.

1000 cu. millimeters$^{(cu\ mm)}$ = 1 cu. centimeter$^{(cu\ cm)}$ = .000001 of a cu. meter
1000 cu. centimeters = 1 cu. decimeter$^{(cu\ dm)}$ = .001 "
1000 cu. decimeters = 1 cu. meter$^{(cu\ m)}$.

113. In measuring wood the cubic meter is called a *ster*$^{(st)}$ (pronounced *stair*).



1 cubic centimeter is the contents of 1 milliliter of water which weighs one gram.

114. A rectangular solid whose length, breadth and thickness are equal to each other is called a *cube*.

115. From the last oral exercise it is evident that when the length, breadth and thickness of a cube or other rectangular solid are given in the *same unit* of length, the product of these three numbers will be its volume in *cubic units* of the same name.

116. ORAL.

1. How many cubic decimeters in a block of ice 10^{dm} long, 10^{dm} wide and 10^{dm} thick? In a block 10^{dm} long, 5^{dm} wide and 3^{dm} thick?

2. A cellar 15^{m} long, 10^{m} wide and 3^{m} deep would hold how many cubic meters?

3. How many cubic meters of air in a room 6^{m} long, 5^{m} wide and 3^{m} high?

4. A room 6^{m} long and 5^{m} wide contains $90^{cu\ m}$ of air. What is its height? Another which contains $140^{cu\ m}$ is 7^{m} long and 4^{m} high. What is its width?

5. What must be the length of a block 5^{dm} wide and 4^{dm} thick to contain $400^{cu\ dm}$?

EXERCISE LIII.

1. In $80.4^{cu\ m}$ how many cubic centimeters?

2. Reduce $14.03^{cu\ m}$ to cubic millimeters.

3. Express $9837216^{cu\ mm}$ in cubic meters.

4. How many cubic centimeters in a bar of iron 12^{m} long, 5^{cm} wide and 2^{cm} thick?

5. In a stick of timber 8.2^{m} long, $.425^{m}$ broad and $.412^{m}$ thick, there are how many cubic meters?

6. $1^{cu\ m}$ is how many times as large as $1^{cu\ cm}$?

7. A bin 5^{m} long, 4.4^{m} wide and 2^{m} high, would contain how many cubic meters of coal? How many sters of wood?

8. In a school-room 9.5^{m} long, 7.2^{m} wide and 3.2^{m} high there are 42 pupils. How many cubic meters of air are there to each pupil?

9. How many cubic meters of air does your school-room contain? How many cubic meters does it allow for each pupil present to-day?

10. How many cubic meters of earth must be removed in digging a cellar 10.25^{m} long, 7.2^{m} wide and 2.6^{m} deep?

11. What is the cost of digging a cellar $8^{m} \times 6^{m} \times 2.25^{m}$ at 75 cents per cubic meter?

12. In a pile of wood 13^{m} long and 2.5^{m} wide and 2^{m} high there are how many cubic meters or sters?

13. What is the height of a room 9^{m} long and 6.5^{m} wide which contains $234^{cu\ m}$ of air?

14. What is the length of a pile of wood 4.1^{m} wide and 3.1^{m} high which contains 264.368^{st}?

15. What is the value of a pile of wood 5.4^{m} long, 2.6^{m} wide and 1.5^{m} high, at \$1.50 per ster?

MEASURES OF CAPACITY.

117. In measuring liquids, grain, etc., the cubic decimeter is called a *liter*.

118.

10 milliliters(ml)	= 1 centiliter(cl)	= .01 of a liter.
10 centiliters	= 1 deciliter(dl)	= .1 " "
10 deciliters	= 1 **liter**(l)	
10 liters	= 1 dekaliter(Dl)	= 10 liters.
10 dekaliters	= 1 *hektoliter*(Hl)	= 100 "
10 hektoliters	= 1 kiloliter(Kl)	= 1000 " = $1^{cu\ m}$.

EXERCISE LIV.

1. In 18.4^{Kl} how many liters?

2. In 72.43^{l} how many centiliters?

3. How many liters of oil will a tank hold which is 1.4^{m} long and $.8^{m}$ broad and deep?

4. A cistern, each side of which is 24.8^{m} long and whose depth is 6.5^{m} will hold how many kiloliters of water?

5. What is the value of a puncheon of molasses containing $1^{Kl}\ 3^{Hl}\ 5^{l}$ at 8 cents per liter?

6. A tap from which water runs at the rate of 50^{l} per minute would deliver how many kiloliters in an hour?

7. How long would the tap mentioned in the last question be in emptying a cistern 4^{m} square and 2.5^{m} deep?

8. What is the value of 400^{ml} of sulphuric acid at 30 cents a liter?

9. A cask of oil containing 3.5^{Hl} was bought for $10 and sold at 4 cents per liter? What was the gain?

10. When a cask of liquid ammonia containing 3.65^{Hl} costs $146, what is the value of 9.15^{l}?

MEASURES OF WEIGHT.

119. The unit of weight is the *gram* and its weight is the weight of a cubic centimeter of distilled water at the temperature of its greatest density, 4° C.

120. TABLE.

10 milligrams(mg)	= 1 *centigram*(cg)	= .01 of a gram	= .15432 grains.	
10 centigrams	= 1 decigram(dg)	= .1 " "	= 1.5432 "	
10 decigrams	= 1 **gram**(g)	=	15.432 "	
10 grams	= 1 dekagram(Dg)	= 10 grams	= .35273	ounces av.
10 dekagrams	= 1 hektogram(Hg)	= 100 "	= 3.5273 "	"
10 hektograms	= 1 *kilogram*(Kg)	= 1000 "	= 2.20462	pounds av.
1000 kilograms	= 1 *metric ton*(T)	= 1000000 "	= 2204.62125 "	"

121. Each of the tables may be written and used in this way:

1000 milligrams	= 1 gram
100 centigrams	= 1 "
10 decigrams	= 1 "
10 grams	= 1 dekagram.
100 "	= 1 hektogram.
1000 "	= 1 kilogram.

122. A liter of water weighs a kilogram.

123. A cubic meter of water weighs a metric ton.

EXERCISE LV.

1. Reduce 17.4^{Kg} to grams.

2. Reduce 68.3^{g} to centigrams.

3. In sixty-three million grams how many metric tons?

4. What would a barrel of sugar weighing 132^{Kg} be worth at 2 cents per 100^{g}?

5. What is the weight of water in a rectangular vessel 75^{cm} long, 50^{cm} wide and 45^{cm} deep?

6. How many kilograms does 784.6^{l} of water weigh?

7. Write in grams: 5^{Kg} 7^{Hg} 4^{Dg} 9^{dg} 25^{mg}.

8. When 15.4^{T} of coal is equally divided among 50 poor people, how many kilograms does each get?

9. Find the weight of water in a cistern 4.2^{m} long, and 3.6^{m} broad and deep.

10. What must be paid for 175^{mg} of quinia at $0.80 per gram?

124. EQUIVALENTS IN CANADIAN MEASURES.

1 meter	= 39.37 + in., or about 1 yd. 3⅜ in.
1 kilometer	= .62137 + mi., or about ⅝ of a mile.
1 square meter	= 1.196 + sq. yd., or about 10¾ sq. ft.
1 ar	= 119.6 + sq. yd., or about 4 sq. rd.
1 hektar	= 2.471 + A., or about 2½ A.
1 cubic meter	= 1.308 + cu. yd., or about 35⅓ cu. ft.
1 liter	= .8804 + imperial quarts, or about 1¾ pints.
1 hektoliter	= 2.751 + bushels, or about 2 bu. 3 pks.
1 gram	= 15.432 + gr., or about 15½ gr.
1 kilogram	= 2.2046 + lb., or about 2¼ lb. (avoirdupois).
1 metric ton	= 2204.6 + lb., or about 1 T. 200 lb.

1 yd. = .9144^{m} + . 1^{m} = 1.0936 + yd.

It will sometimes be found convenient to use the reciprocals of those numbers. They are found thus: 1 qt. $= \frac{1^{l}}{.8804} = 1.136^{l}$.

1 mi. $= \frac{1^{Km.}}{.62137} = 1.60935^{Km}$, or 1.61^{Km}. 1 bu. $= \frac{1^{Hl}}{2.751} = .3635^{Hl.}+$

EXAMINATION PAPER. NO. 11.

(TIME, 1 HOUR. VALUE OF EACH QUESTION, 10.)

1. Express in meters the sum of 88^{cm}, 4.6^{Km}, 133^{m}, 495^{mm}, 50.14^{m}.

2. From the sum of $8.52^{m} + 41.7^{cm}$ take the sum of $.0074^{Km}$ and 1.537^{m}.

3. When 3.15^{m} of cloth is worth \$4.20 what is the value of 85^{cm}?

4. A girl who lives 3.42^{Km} from school is 2.7^{Km} from the post office. How much further must she walk for dinner than when she goes to mail a letter?

5. Simplify $(6.35^{m} \times 29) - (4.8^{Km} \div 32)$.

6. What is the cost of preparing a blackboard 11.375^{m} long and 1.4^{m} wide at \$0.12 per square meter?

7. A grocer bought a ham weighing 8.5^{Kg} for \$1.70 and is selling it at a rate which will give him a profit of \$0.68 on the whole. What must I pay him for 1.75^{Kg}?

8. How do you subtract one vulgar fraction from another?

9. Give in meters the length of the longest panel which can be used in fencing a field 4151^{m} long and 1666^{m} wide, all the panels to be of equal length.

10. If your garden contains 893.75^{qm} and your neighbor's 769.25^{qm}, how many ars are in both?

EXAMINATION PAPER. No. 12.

(TIME, ONE HOUR. VALUE OF EACH QUESTION, 10).

1. Give the weight in grains of a *gram*, a decigram and a milligram, and the weight in pounds of a *kilogram* and of a metric ton.

2. Taking the weight of 1 liter of air to be 1.293^{g}, what is the weight of $1^{cu\ m}$ of air?

3. Simplify $(72.15^{m} + 19.89^{m} - 48.766^{m}) \div 77$.

4. Give an example of your method of finding the G. C. M. of two numbers. Use numbers larger than 100.

5. What weight of air and what weight of water will a vessel $\frac{1}{2}$ of a meter in length and $\frac{1}{3}$ of a meter in width and height hold?

6. What is the value of a pile of wood 263^{m} long, 3.3^{m} wide and 2.5^{m} high at 50 cents per ster?

7. A man bought a farm containing 50^{Ha} of land for $2400. He sold one field 400^{m} long and 200^{m} wide for $660 and the remainder at cost. What was the gain?

8. What will 2.6^{Kg} of opium cost at 8 cents per gram?

9. What is the value of 2.25^{Hl} of petroleum at $.0325 per liter?

10. What does a barrel of flour weigh in kilograms?

EXAMINATION PAPER. No. 13.

(Time, One Hour. Value of Each Question, 10).

1. 20 soldiers are digging a trench which they can finish in 12 days. After they have been engaged in the work 4 days the engineer receives orders to have the work done in 5 days more. How many more men must be employed?

2. At the rate of 3^{Km} an hour how long will it take a boy to walk 11000^{m}?

3. Draw on your paper a diagram which is 5^{cm} square. How many sq. centimeters does it contain? Sq. millimeters?

4. How many liters of oats can be put into a bin 6^{m} long, 3.5^{m} wide and 2.7^{m} deep?

5. What is the value of $\frac{3}{5}$ of $1\frac{1}{4}$ of 4^{Kg} of potash when 750^{g} can be purchased for 6 cents?

6. How would you find the average of several numbers?

7. How many meters of fence will it take to enclose a field 4^{Hm} long and 8^{Dm} wide?

8. How many ars in a field 150^{m} long and 80^{m} wide?

9. Iron is about 7.8 times as heavy as the same bulk of water. What is the weight of a bar 8^{m} long, 5^{cm} wide and 4^{cm} thick?

10. From a farm containing 100^{Ha} there was sold 64^{Ha} 85^{a}. What is the value of the remainder at $65 a hektar?

COMMON WEIGHTS AND MEASURES.

MEASURES OF LENGTH.

125.

12	inches (in.)	=1 foot (ft.)
3	feet	=1 yard (yd.)
$5\frac{1}{2}$	yards	=1 rod or pole (rd.)
40	rods	=1 furlong (fur.)
8	furlongs	=1 mile (mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

The *knot*, a nautical mile, is 6086.7 feet.

A *fathom* is 6 feet.

The *hand*, a term used in measuring horses, is equal to 4 inches.

A *span* = 9 inches, a *common cubit* = 18 inches and a *sacred cubit* = 21.888 inches.

SURFACE MEASURES.

126.

144	square inches (sq. in.)	=1	square foot (sq. ft.)
9	" feet	=1	" yard (sq. yd.)
$30\frac{1}{4}$	" yards	=1	" rod (sq. rd.)
160	" rods	=1	acre (A.)
640	acres	=1	square mile (sq. mi.)

1 sq. mi. = 640 A. = 102400 sq. rd. = 3097600 sq. yd. = 27878400 sq. ft. = 4014489600 sq. in.

Land surveyors use a *chain* 22 yards long, divided into 100 equal parts called *links*. A link is 7.92 in.

10000	square links	=1 square chain (sq. ch.)
10	" chains	=1 acre.

An *engineer's chain* is 100 feet long and consists of 100 *links*. Shingling, roofing, etc., are commonly estimated by the *square*. Each side of the *square* is 10 feet long, containing 100 sq. ft.

MEASURES OF VOLUME.

127.

1728 cubic inches = 1 cubic foot (cu. ft.)
27 " feet = 1 " yard (cu. yd.)

A pile 8 ft. long, 4 ft. wide and 4 ft. high, equal to 128 cu. ft., is called a *cord*. The cord is used in measuring firewood and rough stone. A *perch* of stone or masonry is $24\frac{3}{4}$ cu. ft.

MEASURES OF CAPACITY.

128. This measure is used in measuring oil, molasses, milk, water and other liquids, and such articles as grain, roots, fruit, salt, etc.

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
2 gallons = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

The legal, or "Imperial" bushel of Canada contains 8 Imperial gallons, and 1 Imperial gallon is equal to 277.274 cubic inches. The United States liquid gallon contains 231 cubic inches.

129. The "Weights and Measures" Act of the Canadian Parliament of 1873 fixed the number of pounds to the bushel of each article named below as follows:

Oats ..... 34 lb.	Flax Seed ..... 50 lb.	Beans ..... 60 lb.
Barley ..... 48 lb.	Corn ..... 56 lb.	Peas ..... 60 lb.
Buckwheat ... 48 lb.	Rye ..... 56 lb.	Clover Seed ... 60 lb.
Timothy Seed.. 48 lb.	Wheat ..... 60 lb.	Potatoes ..... 60 lb.

MEASURES OF WEIGHT.

(AVOIRDUPOIS WEIGHT).

130.

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
100 pounds = 1 hundred weight (cwt.)
20 hundred weight = 1 ton.
A long ton = 2240 lbs.

1 T. = 20 cwt. = 2000 lb. = 32000 oz. = 512000 dr.

Everything except precious metals, jewels and medicines is weighed by Avoirdupois weight.

TROY OR JEWELLERS' WEIGHT.

131.

24 grains (gr)	=1 pennyweight (dwt.)
20 pennyweights	=1 ounce (oz.)
12 ounces	=1 pound (lb.)

APOTHECARIES OR DRUGGISTS' WEIGHT.

132.

20 grains(gr).	=1 scruple (sc. or ℈ .)
3 scruples	=1 dram (dr. or ʒ .)
8 drams	=1 ounce (oz. or ℥ .)
12 ounces	=1 pound (lb.)

Drugs are bought and sold by avoirdupois weight although compounded by apothecaries weight. The latter weight, however, will soon, even in pharmacy, give way to avoirdupois weight, which in turn will be replaced by the metric system.

1 lb. avoirdupois = 7000 grains.

1 lb. troy or apothecaries = 5760 grains.

MEASURES OF TIME.

133.

60 seconds(sec.)	=1 minute (min),
60 minutes	=1 hour (hr.)
24 hours	=1 day (da.)
7 days	=1 week (wk.)
365 days	=1 common year (yr.)
366 days	=1 leap year.

The leap years are those whose dates are exactly divisible by 4, except in the case of even hundreds; but these to be leap years must be exactly divisible by 400. Thus 1900 will not be a leap year, but 1904 and 2000 will.

The year is divided into 12 months. Four of them have 30 days each, seven have 31 days each, while February has 28 in common years and 29 in leap years.

Thirty days have September,
April, June and November.
All the rest have thirty-one
Save February, twenty-eight alone;
But leap-year, coming once in four,
Gives February one day more.

The civil day begins and ends at midnight.

The earth revolves round the sun in 365 da. 5 hr. 48 min. 46 sec.

MISCELLANEOUS TABLE.

134.

12 articles	= 1 dozen.	200 lbs. pork or beef	= 1 barrel.
12 dozen	= 1 gross.	24 sheets of paper	= 1 quire.
12 gross	= 1 great gross.	20 qu. or 480 sheets	= 1 ream.
20 articles	= 1 score.	2 reams	= 1 bundle.
196 lbs. of flour	= 1 barrel.	2 bundles	= 1 bale.

ENGLISH MONEY.

135.

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£).

Farthings are generally written as a fraction of a penny. 1, 2 or 3 farthings are written $\frac{1}{4}$d, $\frac{1}{2}$d, $\frac{3}{4}$d.

COMPOUND NUMBERS.

136. EXAMPLE 65.

(1).

738428
249319
518428
276327
7214
1789716

(2).

$7384.28
2493.19
5184.28
2763.27
72.14
$17897.16

(3).

7384.28m
2493.19
5184.28
2763.27
72.14
17897.16m

(4).

Tons.	20 Cwt.	100 Lbs.	16 Oz.	16 Dr.
73	8	4	2	8
24	9	3	1	9
51	8	4	2	8
27	6	3	2	7
	7	2	1	4
176	18	16	10	4

(5).

Miles.	8 Fur.	40 Po.	5½ Yds.	3 Ft.	12 In.
7	3	8	4	2	8
2	4	9	3	1	9
5	1	8	4	2	8
2	7	6	3	2	7
		7	2	1	4
18	0	1	$2\frac{1}{2}$	2	0
			$\frac{1}{2}$ =	1	6
18	0	1	3	0	6

In the first three examples we find that the abstract numbers and the numbers representing dollars and meters are added in exactly the same way.

On adding the first column we get 36. But as 10 in this column makes 1 in the next to the left we find the number of 10's in 36. There are 3 10's and 6 over. We set down the 6 and add the 3 to the next column. The second column gives 11, which being divided by 10 gives 1 and 1 over. The third column gives 17 or 1 and 7 over. We continue the process to the end, dividing the sum of each column by 10, *because 10 in any one of these columns makes 1 in the next to the left.*

Whether we read the numbers in example **3** as $7^{Km}\ 3^{Hm}\ 8^{Dm}\ 4^{m}\ 2^{dm}\ 8^{cm}$ or as 7384.28 meters the adding is done in exactly the same way.

In example **4** we find the sum of the first column to be 36 drams. Now as 16 dr. make 1 oz., we divide by 16, getting 2 oz. and 4 dr. over. We set down the 4 dr. and add the 2 oz. to the column of ounces.

On adding the second column we get 10 oz., which is less than 1 lb., as there are 16 oz. in 1 lb., so we set down the 10 oz.

On adding the third column we get 16 lbs., which, being less than 1 cwt., we set down.

The fourth column gives 38 cwt. We divide the 38 cwt. by 20 cwt. in 1 ton and we get one ton and 18 cwt. We set down the 18 cwt. and add 1 ton to the next column.

In the next example the sum of the first column is 36 inches. As 12 of this column make 1 in the next, or 12 inches make 1 foot, we divide by 12, setting down what is over. In the second column we divide by 3 the number of feet in 1 yard and set down the remainder 2. The sum of the third column we divide by $5\frac{1}{2}$ and get a remainder of $2\frac{1}{2}$. The sum of the fourth column we divide by 40, and the sum of the fifth column we divide by 8.

The $\frac{1}{2}$ yard is equal to 1 ft. 6 in. and we add it to the 2 yds. 2 ft. 0 inches as such. This gives 3 yds. 0 ft. 6 in.

ADDITION.

137. EXERCISE LVI.

(1).

tons.	cwt.	lb.	oz.
17	13	64	12
9	15	8	8
	5	40	15
25	0	38	4

(2).

£.	S.	D.
10	14	6
9	15	$7\frac{1}{2}$
8	6	9
50	19	$11\frac{3}{4}$

(3).

miles.	fur.	po.	yds.	ft.	in.
3	5	35	3	2	11
20	4	18	4	0	10
	7	39	0	1	5
17	0	0	2	2	9

(4).					(5).				(6).				
bu.	pk	gal.	qt.	pt.	days.	hours.	min.	sec.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
18	3	1	2	1	137	16	10	42	110	75	28	8	112
24	2	0	3	0	75	20	27	15	68	100	20	7	58
40	0	1	0	1	7	10	55	48	902	155	29	1	140
27	2	1	2	1	40	9	30	30	88	70	15	5	33

7. Find the sum of 17 A. 95 sq. rd. 15 sq. yd. 135 sq. in.; 302 A. 45 sq. rd. 4 sq. ft. 60 sq. in.; and 10 A. 105 sq. rd. 25 sq. yd. 8 sq. ft.

8. Add 175^{m}, 11.52^{m}, 86.4^{m}, 753^{cm}, and 235^{m}.

9. Add 30 tons, 17 cwt.; 1 ton 15 cwt. 86 lb.; 19 cwt. 96 lb. 14 oz.; 55 lb. 11 oz. 13 dr.; 3 tons 75 lb. 9 oz. 12 dr.

10. Add 49 cu. yd. 15 cu. ft. 612 cu. in.; 17 cu. ft. 500 cu. in.; 841 cu. yd. 675 cu. in.; 53 cu. yd. 4 cu. ft. 754 cu. in.; 99 cu. ft. 1315 cu. in.

NOTE.—Let the teacher place on the board at least 20 questions similar to the foregoing.

MULTIPLICATION.

138. EXAMPLE 66. Multiply the kilograms and also the tons, etc., given below by 29.

$$\begin{array}{r} 7849.65^{Kg} \\ 29 \\ \hline 7064685 \\ 1569930 \\ \hline 227639.85^{Kg} \end{array}$$

tons.	cwt.	lb.	oz.	dr.
78	4	9	6	5
				29
2267	18	72	7	1

When 5 dr. are multiplied by 29 we get 145 dr. which we divide by 16 dr. in 1 oz. This gives us 9 oz. and 1 dr. We set down the 1 dr. Then 6 oz. $\times 29 + 9$ oz. $= 183$ oz. 183 oz. $\div 16 = 11$ lb. 7 oz. Set down 7 oz. 9 lb. $\times 29 + 11$ lb. $= 272$ lb. 272 lb. $\div 100 = 2$ cwt. 72 lb. 4 cwt. $\times 29 + 2$ cwt. $= 118$ cwt. 118 cwt. $\div 20 = 5$ tons 18 cwt. 78 tons $\times 29 + 5$ tons $= 2267$ tons.

EXERCISE LVII.

1. Multiply £25 12s. 6d. by 9.

2. Multiply the sum of £3 " 18s. " $7\frac{1}{2}$d. and £8 14s. $4\frac{3}{4}$d. by 24.

3. Multiply the sum of 54.62^{Kg}, 846.4^{g}, 656^{mg} by 12.

4. Multiply 35 A. 74 sq. rd. 27 sq. yd. 4 sq. ft. 66 sq. in. by 36.

5. If a man walks 4 mi. 5 fur. 35 rd. each morning and evening for a week what distance does he walk?

6. A grocery wagon contained a barrel of flour, a bushel of barley, 1 peck of peas, $1\frac{1}{2}$ bushel of potatoes, $5\frac{1}{2}$ lbs. of tea, 34 lbs. of sugar, 6 oz. soda and 4 oz. pepper. What weight is carried by another wagon which carried just 7 times this weight?

7. A boy went 14 times in one day for water to a spring which was 24 rd. 4 yd. from the house, and he carried home 2 gal. 1 pt. each time. What distance did he travel and how much water did he carry?

SUBTRACTION.

139.

EXAMPLE 67.

A.	sq. rd.	sq. yd.	sq ft.	sq. in.
403	0	25	6	58
223	104	27	7	85
179	55	$27\frac{1}{4}$	7	117
			2	36
179	55	28	1	9

EXAMPLE 68.

mi.	fur.	rd.	yd.	ft.	in.
10	0	0	0	0	0
7	3	26	2	1	9
2	4	13	$2\frac{1}{2}$	1	3
				1	6
2	4	13	2	2	9

As we cannot take 85 sq. in. from 58 sq. in. we take 1 sq. ft., which is equal to 144 sq. in., from the 6 sq. ft. and add it to 58 sq. in. Then we take 85 sq. in. from 202 sq. in. and get a remainder of 117 sq. in.

Then as we cannot take 7 sq. ft. from 5 sq. ft. we take 1 sq. yd., or 9 sq. ft., from the 25 sq. yds. and add it to the 5 sq. ft. Then 7 sq. yds. from 14 sq. yds. leave 7 sq. yds.

Again, as we cannot take 27 sq. yds. from 24 sq. yds., and as there are no sq. rods we take 1 A. or 160 sq. rd. from the 403 A. Then we take one of these sq. rods, or $30\frac{1}{4}$ sq. yds., and add them to our 24 sq. yds., and we subtract 27 sq. yds. from $54\frac{1}{4}$ sq. yds. and get $27\frac{1}{4}$.

Then 104 sq. rds. from 159 sq. rds. leave 55 sq. rds., and 223 A. from 402 A. leave 179 A.

The $\frac{1}{4}$ sq. yd. is equal to 2 sq. ft. 36 in., which we add to the feet and inches as such.

EXERCISE LVIII.

1. A man who had \$212.50 paid a bill of \$136.75. How much money had he left?

2. A man who had £41 5s. $3\frac{1}{4}$d. paid a bill of £22 7s. $3\frac{3}{4}$d. How much money had he left?

3. A man who had to dig a drain 104.25^{m} long got 66.18^{m} finished. How much remained to be done?

4. A man who had to dig a drain 104 yd. 2 ft. 5 in. long got 66 yd. 1 ft. 8 in. done. How much remained to be done?

5. From 1 ton take 3 cwt. 47 lb. 9 oz.

6. From 45 A. 120 sq. rd. take 32 A. 75 sq. rd. 19 sq. yd. 6 sq. ft.

7. From 3^{Km} take 1899.27^{m}.

8. From 20 bu. 1 pk. 1 gal. 1 pt. take 14 bu. 3 pk. 3 qt.

9. From 7 mi. 7 fur. 39 rd. 5 yd. 2 ft. 11 in. take 8 mi.

10. Express in words any difference between the subtraction of compound and of ordinary numbers.

140. COMPOUND DIVISION.

EXAMPLE 69. Divide $\overset{\text{ton.}}{83}$ " $\overset{\text{cwt.}}{12}$ " $\overset{\text{lb.}}{45}$ " $\overset{\text{oz.}}{4}$ into 26 equal parts.

$$
\begin{array}{r|l}
26 & \overset{\text{ton.}}{83} \text{ " } \overset{\text{cwt.}}{12} \text{ " } \overset{\text{lb.}}{45} \text{ " } \overset{\text{oz.}}{4} \\
\hline
 & \overset{\text{ton.}}{3} \text{ " } \overset{\text{cwt.}}{4} \text{ " } \overset{\text{lb.}}{32} \text{ " } 8\tfrac{2}{13}
\end{array}
$$

78

5

20

112 cwt.

104

8

100

845 lb.

78

65

52

13

16

82

13

212 oz.

208

4

26 = 2/13

83 tons divided into 26 equal parts give 3 tons and 5 tons of a remainder. We bring our remainder, 5 tons, to cwt. by multiplying by 20, or the number of cwt. in 1 ton, and we add in the 12 cwt. Our next remainder, 8 cwt. = 800 lb., and when we add the 45 lb. we have 845 lb. for our third dividend.

The remainder 13 lbs. = 13 × 16 or 208 oz. After the 4 oz. are added we get 212 oz., our last dividend.

EXERCISE LIX.

1. Divide 21 hr. 42 min. 30 sec. into 6 equal parts.

2. Divide 27 mi. 5 fur 28 rd. 2 yd. 2 ft. 10 in. into 12 equal parts.

3. Divide 250 bu. 3 pk. 1 gal. 1 qt. into 18 equal parts.

4. What is the average number of miles, furlongs, rods, etc., travelled each hour by a man who walked 20 mi. in 6 hours?

5. When 2 tons 16 cwt. 75 lb. of meal is divided equally among 35 poor people, what is the share of each?

6. Divide a web of 96.435^{m} of cloth into 15 equal parts.

7. What distance per hour would a man require to walk in order to go to a town 10 mi. 6 fur. 28 rd. distant, and return in 7 hours?

8. If £280 5s. 9½d. is paid for 97 tons of lead, what is the price of 1 ton?

9. 451 tons of copper ore will purchase 8008 tons, 17 cwt. 37 lb. 10 oz. of iron ore. How much iron ore will 1 ton of copper ore purchase?

10. When 98 acres produce 1800 bu. 3 pk. 1 gal. 3 qt. what is the average yield per acre?

REDUCTION.

141. It is often necessary to reduce or change numbers from one name or denomination to another.

EXAMPLE 70. Reduce £27 13s. 8¾d. to farthings.

£27 13s. 8¾d.
20
553 shillings.
12
6644 pence.
4
26579 farthings.

To reduce £27 to shillings we multiply by 20, because each pound is equal to 20 shillings. To the product we add the 13s.

To bring the 553s. to pence we multiply by 12, because each shilling equals 12 pence. We add in the 8d. To bring the pence to farthings we multiply by 4, as 1d. = 4 far. We add in the 3 far.

EXAMPLE 71. Reduce 26579 farthings to pounds, etc.

4|26579 far.
12|6644¾ pence.
20|553s. 8¾d.
£27 13s. 8¾d.

To bring the farthings to pence we divide by 4, because 4 far. are required to make 1d. The remainder is farthings. To bring pence to shillings we divide by 12, because 12d. are equal to only 1s. The remainder is pence. To bring shillings to pounds we divide by 20, because 20s. make £1. The remainder is shillings.

EXAMPLE 72. In 17 T. 17 cwt. 46 lbs. 11 oz. 9 dr., how many drams?

17 T. 17 cwt. 46 lb. 11 oz. 9 dr.
20
357 cwt.
100
35746 lb.
16
214487
35746
571947 oz.
16
3431691
571947
9151161 dr.

EXAMPLE 73. In 9151161 dr. how many tons, etc.

16|9151161 dr.
16|571947 oz. 9 dr.
100|35746 lb. 11 oz.
20|357 cwt. 46 lb.
17 T. 17 cwt. 46 lbs. 11 oz. 9 dr.

142. Examples 71 and 73 are of *Reduction Ascending;* 70 and 72 are of *Reduction Descending.*

EXERCISE LX.

1. Reduce £3 9s. 6¼d. to farthings.
2. Reduce £309 19s. 11¾d. to farthings. Prove.
3. Reduce 2000 far. to pounds, etc.
4. Reduce 768005 far. to pounds, etc.
5. Express 10000 pence in pounds, etc. Prove.
6. Express £963 18s. in pence. Prove.
7. Bring 15 cwt. 18 lb. to ounces. Prove.
8. Bring 3 T. 29 lbs. to drams.
9. Bring 14 T. 13 cwt. 96 lb. 14 oz. 12 dr. to drams.
10. In 17½ cwt. how many ounces?
11. In 9 da. 15 hr. 48 m. how many minutes? Prove.

12. How many pints in 27 bu. 3 pk. 1 gal. 3 qt.?

13. How many bushels, etc., in 7363 pints? Prove.

14. In 5 wk. 4 da. 21 hr. how many seconds?

15. A solar year is 365 da. 5 hr. 48 min. 46 sec. How many seconds?

16. How many tons, etc., in 100044 ounces? Prove.

17. How many seconds from 9 o'clock Monday to 16 o clock the following Friday?

18. In 617^{m} how many centimeters?

19. How many tons, etc., in 8007 lb. 12 oz. 10 dr.?

20. How would you reduce farthings to pounds?

EXAMPLE 74. Express in inches:

m. fur. rd. yd. ft. in.
37 " 6 " 35 " 2 " 1 " 11

8
302 fur.
40
12115 rd.
$5\frac{1}{2}$
$60571\frac{1}{2}$
60577
66634 yd. 1 ft. 6 in.
3
199904 ft.
12
2398865 in.

EXAMPLE 75. How many acres in 20067725 sq. inches?

144 | 20067725 sq. in.
9 | 139359 sq. ft. 29 sq. in.
$30\frac{1}{4}$ | 15484 sq. yds. 3 sq. ft.
4
121 | 61936
160 | 511 sq. rd. $\frac{105}{4} = 26\frac{1}{4}$ sq. yd.
3 A. 31 sq. rd. $26\frac{1}{4}$ sq. yd. 3 sq. ft. 29 sq. in.
$\frac{1}{4}$ " = 2 " 36 "
3 A. 31 sq. rd. 26 sq. yd. 5 sq. ft. 65 sq. in.

EXERCISE LXI.

1. Express 2398865 in. in miles, etc.
2. Express 20 mi. 5 fur. 2 yds. in inches.
3. Express $788.3^{m.}$ in millimeters.
4. Express 96012^{g} in kilograms.
5. In 17280 grains how many pounds troy?
6. In 28000 grains how many pounds troy? Apothecaries? Avoirdupois?
7. How many seconds from 21 o'clock on the first day of the month to a quarter past 9 on the 15th day?
8. In 3 A. 31 sq. rd. 26 sq. yd. 5 sq. ft. 65 sq. in. how many square inches?
9. In $2^{oz.}$ how many grams? How many grains?
10. How many acres, etc., in 10000000 sq. inches?
11. Bring 230604 qts. to bushels. Prove work.
12. In England there are 50535 sq. miles, in Wales, 8125 square miles, in Scotland, 29167 sq. miles. How many acres in the whole island of Great Britain?
13. Reduce 864^{m} to inches.
14. How many meters in 82677 inches?
15. Give in inches the perimeter (measure around) of a school room which is 28 ft. 6 in. long and 22 ft. 9 in. wide. Also in meters.
16. How many miles in 86511 feet?
17. Into how many building lots, each 24 ft. wide and 100 ft. deep, can an acre field be divided?
18. Bought 8 cwt. 56 lb. of sugar at $4\frac{1}{2}$ cents per lb., and sold it at $5\frac{1}{4}$ cents. What was the gain?
19. Bought 3 T. 12 cwt. 84 lbs. of leather at $14\frac{5}{8}$ cents per lb., and sold it at $15\frac{3}{16}$ cents. What did I gain?
20. How often would a carriage wheel, 12 ft. 6 in. in circumference, turn in going 110 miles?

ANSWERS.

Exercise I.—(Page 6).—**1.** 2,2,2. **2.** 2,5. **3.** 2,2,3. **4.** 2,3,3. **5.** 3,7. **6.** 2,2,2,3. **7.** 2,13. **8.** 3,3,3. **9.** 3,11. **10.** 3,13. **11.** 2,2,2,2,3. **12.** 3,3,17. **13.** 2,3,19. **14.** 3,3,3,3,3. **15.** 2,2,2,5,5. **16.** 2,3,3,5,5. **17.** 2,2,3,5,7. **18.** 2,3,5,23.

Exercise II.—(Page 6).—**1.** 2,2,3,5,5. **2.** 2,3,5,11. **3.** 2,3,5,13. **4.** 2,3,5,17. **5.** 3,3,3, 37. **6.** 7,11,13. **7.** 5,5,41. **8.** 2,2,2,2,2,2,2,2,2,2. **9.** No prime factor. **10.** 2,2,3,5,19. **11.** 2,2,2,137. **12.** 13,89. **13.** 11,97. **14.** 7,13,13. **15.** 5,5,5,13. **16.** 7,293. **17.** 13,293. **18.** 3,3,709. **19.** 2,2,2,3,3,3,3,11. **20.** 2,3,7,11,17. **21.** No prime factor. **22.** 3,3,5,5,5,5. **23.** No prime factor. **24.** 2,2,2,2,2,2,2,2,3,3,7. **25.** 3,7,13,101. **26.** 7,17,61. **27.** 3,3,11,709. **28.** 2,7,13,103. **29.** 101,103. **30.** 23,41,71.

Exercise III.—(Page 8).—**1.** 4. **2.** 3. **3.** 5. **4.** 6. **5.** 8. **6.** 7. **7.** 9. **8.** 11. **9.** 13. **10.** 11. **11.** 12. **12.** 4. **13.** 33. **14.** 42. **15.** 11. **16.** 25. **17.** 144. **18.** 44. **19.** 9. **20.** 6. **21.** 91. **22.** 12. **23.** 40. **24.** 30. **25.** 78. **26.** 11. **27.** 343. **28.** 10. **29.** 7. **30.** 4.

Exercise IV.—(Page 9).—**1.** 19. **2.** 23. **3.** 11. **4.** 37. **5.** 28. **6.** 11. **7.** 7. **8.** Prime. **9.** 270. **10.** 11. **11.** 61. **12.** Prime. **13.** 13. **14.** 113. **15.** 21. **16.** 13. **17.** 42. **18.** 43. **19.** Prime. **20.** 47. **21.** Prime. **22.** 5. **23.** 253. **24.** Prime. **25.** 9. **26.** Prime. **27.** 36. **28.** 4. **29.** 31. **30.** 12140. **31.** 5 feet. **32.** 6 lbs. **33.** 12 feet. **34.** 10 yards. **35.** 60 feet. **36.** $45.

Exercise V.—(Page 11).—**1.** 96. **2.** 144. **3.** 180. **4.** 84. **5.** 144. **6.** 1260. **7.** 1890. **8.** 420. **9.** 210. **10.** 156. **11.** 132. **12.** 204. **13.** 6300. **14.** 17424. **15.** 792. **16.** 120120. **17.** 1377684. **18.** 7560. **19.** $720. **20.** 120 feet. **21.** 12 bushels. **22.** 77. **23.** 202475.

Exercise VI.—(Page 12).—**1.** 4; 504. **2.** 1. **3.** 2520. **4.** 40040. **5.** 170. **6.** 205. **7.** 3. **8.** 1133900. **9.** 27720. **10.** 228150. **11.** 232. **12.** 5950. **13.** 9. **14.** 36. **15.** 114.

Examination Paper, No. 1.—(Page 13).—**1.** $3.75. **2.** $11.10. **3.** 40 lbs. **4.** 11 cents. **5.** 29 cents. **6.** $4500. **7.** $2. **8.** 48. **9.** **10.** 60.

Exercise VII.—(Page 15).—**1.** $0.45. **2.** $5.50. **3.** $4.32. **4.** $25.60. **5.** 84 feet. **6.** $3.20; $4.80; $8. **7.** 1536. **8.** $200. **9.** $36. **10.** 22 men. **11.** 24 men. **12.** 60 days. **13.** 52 **14.** 40 cents. **15.** A. $0.62½; B, $0.25; C, $0.12½.

Exercise VIII.—(Page 16).—**1.** $2.16. **2.** $4.80. **3.** $1680. **4.** $1200. **5.** $150. **6.** $0.84. **7.** $28.80. **8.** 15. **9.** $2.75. **10.** 420; 300.

Exercise IX.—(Page 19).—**1.** $\frac{1}{6}$. **2.** $\frac{4}{7}$. **3.** $\frac{2}{3}$. **4.** $\frac{2}{5}$. **5.** $\frac{3}{4}$. **6.** $\frac{1}{3}$. **7.** $\frac{7}{24}$. **8.** $\frac{5}{6}$. **9.** $\frac{9}{20}$. **10.** $\frac{1}{3}$. **11.** $\frac{97}{101}$. **12.** $\frac{71}{97}$. **13.** $\frac{71}{121}$. **14.** $\frac{124}{437}$. **15.** $\frac{26}{325}$. **16.** $\frac{8}{9}$. **17.** $\frac{5}{6}$. **18.** $\frac{986}{1205}$. **19.** $\frac{181}{120}$. **20.** $\frac{334}{451}$. **21.** $\frac{101}{113}$. **22.** $\frac{361}{487}$. **23.** $\frac{31}{37}$. **24.** $\frac{62}{123}$. **25.** $\frac{29}{37}$. **26.** $\frac{791}{751}$. **27.** $\frac{811}{907}$. **28.** $\frac{241}{461}$. **29.** $\frac{811}{991}$. **30.** $\frac{415}{1942}$.

Exercise X.—(Page 20).—**1.** $\frac{19}{4}$. **2.** $\frac{23}{5}$. **3.** $\frac{51}{8}$. **4.** $\frac{65}{6}$. **5.** $\frac{130}{11}$. **6.** $\frac{15}{8}$. **7.** $\frac{219}{11}$. **8.** $\frac{137}{16}$. **9.** $\frac{632}{23}$. **10.** $\frac{3101}{100}$. **11.** $\frac{199}{100}$. **12.** $\frac{1475}{6}$. **13.** $\frac{268}{21}$. **14.** $\frac{611}{41}$. **15.** $\frac{42824}{365}$. **16.** $\frac{146957}{61}$. **17.** $\frac{904598}{361}$. **18.** $\frac{1474649}{360}$. **19.** $\frac{12903}{11789}$. **20.** $\frac{1590983}{38}$. **21.** $\frac{342032529}{90001}$. **22.** $\frac{112}{16}$; $\frac{400}{40}$. **23.** $\frac{399}{21}$; $\frac{187}{11}$. **24.** $\frac{1296}{72}$; $\frac{210}{14}$.

Examination Paper, No. 2.—(Page 21).—**1.** 550 miles. **2.** 40 men. **3.** $5.00. **4.** 3007. **5.** $52.50. **6.** 9 yards. **7.** Lost $120. **8.** 288. **9.** 81. **10.** 50 cents.

Exercise XI.—(Page 22).—**1.** $3\frac{3}{5}$. **2.** $2\frac{1}{8}$. **3.** $2\frac{3}{5}$. **4.** $5\frac{5}{6}$. **5.** $4\frac{1}{5}$. **6.** 7. **7.** 4. **8.** $1\frac{21}{95}$. **9.** $3\frac{1}{6}$. **10.** $4\frac{4}{23}$. **11.** $81\frac{1}{23}$. **12.** $131\frac{8}{25}$. **13.** $29\frac{53}{234}$. **14.** $390\frac{103}{120}$. **15.** $11413\frac{324}{375}$. **16.** $391\frac{391}{2555}$. **17.** $37\frac{598}{2469}$. **18.** $2\frac{1}{3}$. **19.** $3\frac{341}{2003}$. **20.** 9. **21.** 2002. **22.** $167\frac{277}{4573}$. **23.** $9\frac{1195}{9387}$. **24.** $187\frac{87}{460}$.

Exercise XII.—(Page 23).—**1.** 4. **2.** 80. **3.** 96. **4.** 9. **5.** 13. **6.** 26 cents. **7.** 30 cents. **8.** 27 yards. **9.** 70. **10.** $22\frac{3}{8}$.

Exercise XIII.—(Page 25).—**1.** $\frac{19}{12}$. **2.** $\frac{13}{16}$. **3.** $\frac{19}{32}$. **4.** $\frac{27}{38}$. **5.** $\frac{28}{45}$. **6.** $\frac{42}{72}$. **7.** $\frac{9}{42}$. **8.** $\frac{27}{45}$. **9.** $\frac{39}{59}$. **10.** $\frac{55}{100}$. **11.** $\frac{32}{84}$. **12.** $\frac{44}{55}$. **13.** $\frac{56}{144}$. **14.** $\frac{27}{207}$. **15.** $\frac{11490}{18360}$.

Exercise XIV.—(Page 25).—**1.** $\frac{18}{24}, \frac{20}{24}, \frac{21}{24}$. **2.** $\frac{20}{24}, \frac{21}{24}, \frac{10}{24}$. **3.** $\frac{15}{30}, \frac{21}{30}, \frac{22}{30}$. **4.** $\frac{63}{72}, \frac{30}{72}, \frac{28}{72}$. **5.** $\frac{140}{360}, \frac{285}{360}, \frac{156}{360}$. **6.** $\frac{17}{66}, \frac{21}{66}, \frac{38}{66}$. **7.** $\frac{6}{102}, \frac{26}{102}, \frac{51}{102}$. **8.** $\frac{30}{78}, \frac{9}{78}, \frac{14}{78}$. **9.** $\frac{135}{360}, \frac{252}{360}, \frac{160}{360}, \frac{220}{360}$. **10.** $\frac{96}{360}, \frac{110}{360}, \frac{255}{360}, \frac{190}{360}$. **11.** $\frac{36}{168}, \frac{136}{168}, \frac{6}{168}, \frac{21}{168}$. **12.** $\frac{27}{144}, \frac{46}{144}, \frac{1296}{144}$. **13.** $\frac{1335}{9690}, \frac{2090}{9690}, \frac{6814}{9690}$. **14.** $\frac{24}{4}, \frac{27}{4}, \frac{28}{4}$. **15.** $\frac{165}{30}, \frac{72}{30}, \frac{100}{30}$. **16.** $\frac{231}{4389}, \frac{1353}{4389}, \frac{399}{4389}$. **17.** $\frac{70}{170}, \frac{45}{170}, \frac{46}{170}$. **18.** $\frac{5117}{92106}, \frac{6814}{92106}$.

Exercise XV.—(Page 26).—**1.** $\frac{3}{18}, \frac{3}{8}, \frac{5}{9}, \frac{7}{12}$. **2.** $\frac{2}{5}, \frac{9}{14}, \frac{8}{11}, \frac{9}{12}$. **3.** $\frac{7}{18}, \frac{5}{12}, \frac{13}{30}, \frac{11}{25}$. **4.** $\frac{4}{9}, \frac{13}{24}, \frac{11}{20}, \frac{7}{12}$. **5.** $\frac{17}{64}, \frac{4}{15}, \frac{13}{48}, \frac{7}{24}$. **6.** $\frac{7}{19}, \frac{11}{21}$, or $\frac{16}{24}, \frac{17}{23}$.

Exercise XVI.—(Page 27).—**1.** $\frac{23}{24}$. **2.** $2\frac{1}{8}$. **3.** $\frac{41}{45}$. **4.** $1\frac{3}{4}$. **5.** $2\frac{1}{60}$. **6.** $1\frac{53}{80}$. **7.** $\frac{2113}{2583}$. **8.** $\frac{481}{602}$. **9.** $2\frac{293}{495}$. **10.** $2\frac{103}{300}$. **11.** $3\frac{1}{20}$. **12.** $2\frac{65}{72}$. **13.** $1\frac{81}{88}$. **14.** $1\frac{67}{171}$. **15.** $1\frac{365}{396}$. **16.** $2\frac{3}{4}$. **17.** $\frac{503}{2145}$. **18.** $1\frac{59}{176}$.

Exercise XVII.—(Page 27).—**1.** $14\frac{1}{12}$. **2.** $13\frac{13}{21}$. **3.** $17\frac{5}{8}$. **4.** $11\frac{5}{6}$. **5.** $35\frac{13}{30}$. **6.** $9\frac{11}{16}$. **7.** $5\frac{1}{132}$. **8.** $39\frac{78}{325}$. **9.** $102\frac{5}{19}$. **10.** $160\frac{23}{78}$. **11.** $162\frac{1}{322}$. **12.** $599\frac{1647}{22610}$. **13.** $51\frac{196}{552}$. **14.** $33\frac{1313}{2525}$. **15.** $15\frac{1473}{1872}$. **16.** $18\frac{5}{11}$. **17.** $37\frac{9}{16}$. **18.** $30\frac{79}{325}$.

Exercise XVIII.—(Page 28).—**1.** $\frac{1}{4}$. **2.** $\frac{1}{12}$. **3.** $\frac{11}{24}$. **4.** $\frac{1}{24}$. **5.** $\frac{31}{126}$. **6.** $\frac{5}{48}$. **7.** $\frac{11}{72}$. **8.** $\frac{46}{221}$. **9.** $\frac{153}{233}$. **10.** $\frac{1}{4}$. **11.** $\frac{1}{8}$. **12.** $\frac{1}{7}$. **13.** $\frac{11}{156}$. **14.** $\frac{31}{133}$. **15.** $\frac{17}{42}$. **16.** $\frac{151}{360}$. **17.** $\frac{15}{34}$. **18.** $\frac{303}{1111}$.

Exercise XIX.—(Page 29)—**1.** $1\frac{1}{4}$. **2.** $1\frac{7}{8}$. **3.** $2\frac{1}{16}$. **4.** $1\frac{83}{100}$. **5.** $11\frac{13}{16}$. **6.** $7\frac{91}{132}$. **7.** $7\frac{1}{12}$. **8.** $5\frac{899}{1414}$. **9.** $19\frac{57}{176}$. **10.** $\frac{853}{1848}$. **11.** $2\frac{3}{16}$. **12.** $3\frac{3}{8}\frac{3}{8}\frac{7}{16}$. **13.** $16\frac{7}{176}$. **14.** $151\frac{65}{384}$. **15.** $\frac{88}{91}$. **16.** $\frac{1}{81}\frac{0}{0}\frac{2}{8}\frac{3}{8}$. **17.** $105\frac{7}{176}$. **18.** $79\frac{3}{8}\frac{2}{8}$. **19.** $48\frac{1}{13}\frac{9}{7}\frac{2}{13}$. **20.** $163\frac{1}{11}\frac{8}{7}$. **21.** $105\frac{1}{4}\frac{4}{6}\frac{3}{8}$. **22.** $154\frac{1}{13}\frac{1}{0}$. **23.** 1. **24.** $\frac{9}{1}\frac{8}{6}\frac{1}{6}$. **25.** $130\frac{3}{4}\frac{2}{6}\frac{8}{8}$. **26.** $1599\frac{1}{10}\frac{9}{7}$. **27.** $\frac{11}{45}$. **28.** $\frac{3}{3}\frac{0}{7}\frac{3}{3}\frac{3}{5}$. **29.** $\frac{3}{12}\frac{2}{8}\frac{2}{7}$. **30.** $\frac{2}{2}\frac{1}{7}\frac{0}{6}\frac{1}{3}$.

Exercise XX.—(Page 29).—**1.** $4\frac{7}{8}$. **2.** $19\frac{13}{120}$. **3.** $9\frac{1}{4}\frac{8}{2}$. **4.** 13. **5.** $2\frac{3}{2}\frac{1}{8}\frac{0}{0}$. **6.** $1\frac{2}{3}$.

Exercise XXI.—(Page 29).—**1.** $2\frac{5}{8}$ lbs. **2.** $215\frac{3}{24}$ lbs. **3.** $12\frac{0}{7}\frac{1}{7}$ gallons. **4.** $\$10\frac{2}{4}\frac{9}{8}$. **5.** $144\frac{1}{16}$ acres. **6.** $14\frac{2}{3}$. **7.** $2\frac{3}{8}$. **8.** $\$81\frac{1}{8}$. **9.** $1\frac{1}{2}\frac{1}{1}\frac{7}{5}\frac{1}{8}\frac{1}{8}$. **10.** $\$5\frac{1}{2}\frac{9}{0}$.

Examination Paper, No. 3.—(Page 30).—**1.** $\frac{27}{35}$, $\frac{30}{35}$, $\frac{28}{35}$, $\frac{33}{35}$. **2.** $3\frac{1}{8}$. **3.** $\frac{21}{64}$. **4.** $13\frac{1}{5}$ hours. **5.** $\$0.16\frac{1}{2}$. **6.** \$400. **7.** \$560. **8.** 20 days. **9.** \$135. **10.** 25.

Exercise XXII.—(Page 32).—**1.** $3\frac{3}{4}$. **2** $1\frac{1}{2}$. **3.** $4\frac{1}{8}$. **4.** $4\frac{1}{8}$. **5.** $11\frac{3}{4}$. **6.** $8\frac{2}{11}$. **7.** 3. **8.** 12. **9.** $5\frac{2}{11}$. **10.** $2\frac{1}{2}$. **11.** 9. **12.** 118. **13.** $27\frac{1}{11}$. **14.** $49\frac{1}{2}$. **15.** 9900. **16.** 375. **17.** $237\frac{1}{2}$. **18.** 24.

Exercise XXIII.—(Page 32).—**1.** $\frac{2}{5}$. **2.** $\frac{7}{8}\frac{7}{0}$. **3.** $\frac{11}{66}$. **4.** $\frac{37}{190}$. **5.** $\frac{1}{11}\frac{1}{5}\frac{7}{6}$. **6.** $\frac{1}{190}$. **7.** $\frac{13}{48}$. **8.** $\frac{3}{8}$.

Exercise XXIV.—(Page 33).—**1.** $\frac{9}{32}$. **2.** $\frac{15}{56}$. **3.** $\frac{35}{48}$. **4.** $\frac{56}{81}$. **5.** $\frac{87}{56}$. **6.** $\frac{2}{35}$. **7.** $47\frac{1}{4}$. **8.** $33\frac{33}{88}$. **9.** $\frac{1}{8}$. **10.** $\frac{1}{10}$. **11.** $3\frac{1}{2}$. **12.** $57\frac{3}{7}$. **13.** 1. **14.** 1. **15.** $5\frac{23}{46}\frac{15}{92}$. **16.** $\frac{1}{7}$. **17.** $\frac{6}{66}$. **18.** $\frac{2}{39}$.

Exercise XXV.—(Page 34).—**1.** $\frac{1}{6}$. **2.** $\frac{21}{25}\frac{7}{7}$. **3.** $4\frac{1}{2}$. **4.** $1\frac{1}{6}$. **5.** 1. **6.** $46\frac{17}{18}$. **7.** $33\frac{3}{8}$. **8.** $\frac{50}{3}\frac{3}{0}\frac{3}{3}$. **9.** $\frac{34}{38}\frac{5}{5}$. **10.** 1.

Examination Paper, No. 4.—(Page 34).—**1.** 38 barrels. **2.** 12. **3.** $\frac{2}{67}\frac{5}{24}$. **4.** \$72.00. **5.** \$0.30. **6.** $\$141\frac{7}{10}$. **7.** $10\frac{1}{13}\frac{8}{8}\frac{1}{0}$. **8.** $\frac{1}{18}\frac{9}{8}\frac{8}{0}$. **9.** 177 yards. **10.** $\frac{7}{12}$.

Exercise XXVI.—(Page 34).—**1.** \$0.75. **2.** $\$0.54\frac{3}{8}$. **3.** $\$2.87\frac{9}{16}$. **4.** $17\frac{13}{16}$. **5.** $77\frac{9}{50}$. **6.** $\$12.58\frac{3}{4}$. **7.** $\frac{35}{84}$ acres. **8.** $\frac{27}{40}$. **9.** \$16.00. **10.** $\$1\frac{7}{8}$. **11.** 12 yards. **12.** $3\frac{1}{2}$ cents.

Exercise XXVII.—(Page 36).—**1.** 18. **2.** 22. **3.** 25. **4.** $5\frac{1}{4}$. **5.** $11\frac{13}{17}$. **6.** $1\frac{1}{24}$. **7.** $7\frac{1}{2}$. **8.** $1\frac{7}{11}$. **9.** $71\frac{6}{17}$. **10.** $5\frac{23}{41}$. **11.** $9\frac{3}{16}$. **12.** $1\frac{1}{2}$, $\frac{7}{8}$. **13.** $1\frac{3}{16}$. **14.** $166\frac{2}{3}$. **15.** $4\frac{2}{41}$. **16.** 12 boxes. **17.** 11 pairs. **18** 20 days. **19.** 21 yards. **20.** 14 hours. **21.** 24 bags. **22.** 162. **23.** $1\frac{7}{16}$. **24.** $3\frac{1}{14}$. **25.** $\$\frac{3}{4}\frac{1}{4}$.

Examination Paper No. 5.—(Page 36).—**1.**
2. 16999999910042. **3.** 3, 11, 29, 67. **4.** 42633. **5.** $7\frac{0}{8}\frac{1}{1}$. **6.** \$3000; 300. **7.** \$90. **8.** A \$180, B \$250, C \$200. **9.** $4\frac{0}{3}\frac{9}{2}$. **10.** 20 boxes.

Exercise XXVIII.—(Page 37).—**1.** $\$3.93\frac{3}{4}$. **2.** \$7.50. **3.** $248\frac{5}{8}$ bushels. **4.** \$1320. **5.** 18 acres. **6.** 60 feet. **7.** $5\frac{1}{2}$ bushels. **8.** \$1.76. **9.** \$5.85. **10.** \$18.

Exercise XXIX.—(Page 38).—**1.** 40 eggs. **2.** $8\frac{3}{5}$ feet. **3.** $\frac{5}{6}$. **4** 5 lbs. **5.** $41\frac{1}{11}$ bushels. **6.** \$1350. **7.** $18\frac{3}{4}$ dozen.

Exercise XXX.—(Page 39).—1. $\frac{8}{10}$. 2. $6\frac{3}{5}$. 3. $\frac{1}{8}$. 4. $1\frac{7}{8}$. 5. $1\frac{9}{18}$. 6. $1\frac{11}{12}$. 7. $\frac{44}{48}$. 8. $\frac{84}{88}$. 9. $6\frac{1}{4}$. 10. 30. 11. $2\frac{3}{28}$. 12. $1\frac{41}{63}$. 13. $\frac{175}{176}$. 14. $\frac{80}{231}$. 15. $1\frac{1}{10}$.

Exercise XXXI.—(Page 39) —1. $4\frac{7}{8}$. 2. $\frac{899}{1000}$. 3. $\frac{1}{8}$. 4. $\frac{19}{128}$. 5. $\frac{17}{26}$. 6. $\frac{1}{12}$. 7. $1\frac{11}{25}$. 8. $\frac{88}{963}$. 9. $\frac{1}{4}$.

Exercise XXXII.—(Page 40).—1 $\frac{9}{10}$. 2. $\frac{5}{1008}$. 3. $1\frac{1}{16}$. 4. $\frac{1}{8}$. 5. $\frac{3}{8}$. 6. $\frac{1}{4}$. 7. $\frac{14}{37}$. 8. $\frac{24}{75}$. 9. $\frac{2}{7}$. 10. $\frac{662}{1250}$.

Exercise XXXIII.—(Page 40) —1. 100. 2. 96. 3. 99. 4. 216. 5. 180. 6. 936. 7. 770. 8. $6\frac{3}{4}$. 9. $20\frac{3}{5}$.

Examination Paper, No. 6.—(Page 40).—1. $1089.85. 2. $5000\frac{39}{80}$. 3. 630 yards. 4. $16\frac{3}{4}$. 5. 16 barrels. 6. $5\frac{7}{96}$. 7. $\frac{7}{10}$. 8. 64. 9. $\frac{16}{17}$. 10. $18000.

Exercise XXXIV.—(Page 41).—1. $6\frac{38}{45}$. 2. $\frac{17}{48}$. 3. $\frac{1}{16}$. 4. $1\frac{17}{84}$. 5. $\frac{6}{32}$. 6. $1\frac{2}{57}$. 7. $18\frac{103}{270}$. 8. $\frac{17}{1850}$. 9. $\frac{8875}{10263}$. 10. $\frac{16}{1053}$.

Exercise XXXV.—(Page 42).—1. $0.40; $0.70; $0.45; 0.63\frac{7}{11}$. 2. $1.80; 3.87\frac{1}{2}$; $8.30; 1.83\frac{1}{3}$. 3. $17.75; $3.45; 18.29\frac{7}{7}$ 4. $10.55; 7.68\frac{3}{4}$; $0.01. 5. 0.03\frac{1}{3}$; $6.02; 3.02\frac{3}{4}$.

Exercise XXXVI.—(Page 42).—1. $32. 2. $14.25. 3. 3\frac{13}{18}$. 4. $4.20. 5. 83$\frac{3}{7}$ lbs. 6. 32$\frac{1}{2}$ yards. 7. 18$\frac{13}{15}\frac{1}{7}$ acres. 8. 30 days. 9. $400. 10. 19$\frac{1}{8}$ yards.

Exercise XXXVII.—(Page 43).—1. $500. 2. $4\frac{13}{51}$. 3. $1\frac{13}{14}$. 4. $\frac{5}{12}$; $2\frac{7}{8}$ days. 5. $1\frac{13}{47}$ days. 6. 6. 7. 6. 8. $\frac{1}{16}$. 9. $180. 10. $66\frac{2}{3}$. 11. $400; $600. 12. $1\frac{8}{87}$. 13. $\frac{11}{15}$. 14. $336; A's $56, B's $48. 15. $11\frac{2071}{2184}$. 16. 45\frac{7}{8}$. 17. $\frac{23}{35}$. 18. 8$\frac{1}{2}$ days.

Examination Paper, No. 7.—(Page 44).—1. ———. 2. $64000. 3. 270. 4. 540. 5. 80. 6. A 8.22\frac{2}{7}$; B 4.11\frac{3}{7}$; C 2.05\frac{4}{7}$. 7. $\frac{13}{108}$. 8. 21 barrels and 56 cents over. 9. 200 acres. 10. $2\frac{5}{11}$ days.

Exercise XXXVIII.—(Page 47).—1. Five-tenths; five-hundredths; five-thousandths; five ten-thousandths.

2. Eight ten-thousandths; eight hundred-thousandths; eight millionths.

3. Nine ten-millionths; sixteen hundred thousandths.

4. Thirty-seven hundredths; thirty-seven thousandths; thirty-seven hundred-thousandths.

5. Three hundred sixty-eight ten-thousandths; three hundred sixty-eight hundred-thousandths; three hundred sixty-eight millionths.

6. Four hundred six ten-thousandths; one thousand one-millionths; three thousand six hundred ten-thousandths.

7. Two million thirty-six thousand nine ten-millionths; eight million sixty thousand eleven hundred-millionths.

8. Four thousand one hundred four millionths; eight hundred seven thousand sixteen ten-millionths.

9. Two million twenty-seven thousand sixteen ten-millionths; three million sixty thousand eighteen ten-millionths.
10. Four hundred thousand one hundred sixty-one millionths; four hundred thousand one hundred sixty-one billionths.

Exercise XXXIX.—(Page 47).—1. Six and four tenths or sixty-four tenths; six and forty-four hundredths or six hundred forty-four hundredths; six and four-hundredths or six hundred four hundredths.

2. Sixty-three and eight tenths or six hundred thirty-eight tenths; sixty-three and eight hundredths or six thousand three hundred eight hundredths; sixty-three and eight thousandths or sixty-three thousand eight thousandths.
3. Eight hundred forty-six and eighty-six hundredths or eighty-four thousand six hundred eighty-six hundredths; eight hundred forty-six and eighty-six ten-thousandths or eight million four hundred sixty thousand eighty-six ten-thousandths; three thousand six hundred eighty and forty-six ten-thousandths or thirty-six million eight hundred thousand forty-six ten-thousandths.
4. Eighty-seven and eighty-nine ten-thousandths or eight hundred seventy thousand eighty-nine ten-thousandths; eight hundred seventy-six and one thousand one ten-millionths, or eight billion seven hundred sixty million one thousand one ten-millionths.
5. Nine million six hundred sixty-three thousand seventy-four and eighty-seven thousandths, or nine billion six hundred sixty-three million seventy-four thousand eighty-seven thousandths; one thousand sixty and eight hundred seventy hundred-thousandths, or one hundred six million eight hundred seventy hundred-thousandths.

Exercise XL.—(Page 48).—1. .7; .3; .9; .5. 2. .03; .17; .19; .75.
3. .044; .004; .183. 4. .0004; .0016; .000196. 5. 6.7; 186.19; 26.0014.
6. .07; .044. 7. .0406. 8. .500; .00005. 9. .000200; .00000002.
10. 400.00019. 11. 6400.0000001. 12. 1003001.000000017.

Exercise XLI.—(Page 48).—1. $\frac{8}{10}$. 2. $\frac{13}{100}$. 3. $\frac{146}{1000}$. 4. $\frac{8}{100}$.
5. $\frac{16}{1000}$. 6. $\frac{84}{100000}$. 7. $\frac{3074}{1000000}$. 8. $\frac{88}{100000}$. 9. $\frac{406094}{1000000}$. 10. $\frac{303097}{1000000}$.
11. $\frac{4067}{10000000}$. 12. $\frac{7}{100000000}$. 13. $6\frac{34}{100}$. 14. $17\frac{96}{1000}$. 15. $143\frac{9}{1000}$.
16. $176\frac{504}{1000}$. 17. $6\frac{7}{1000}$. 18. $1\frac{10}{100000}$.

Exercise XLII.—(Page 49).—1. .159. 2. 3.1844. 3. 2.2404.
4. 18.36. 5. 182.2385. 6. 136.917. 7. 80.4444. 8. 71.1104.
9. 145.05. 10. 137.0608. 11. 83.3325. 12. 9597.6818.
13. 51.57 miles. 14. $66.435. 15. 32.675 acres.

Exercise XLIII.—(Page 50).—1. 13.9327. 2. 10.924. 3. 4.859.
4. 16.946. 5. .9991. 6. 5.846. 7. 18.99. 8. .72. 9. 16.996.
10. 34.0401. 11. 999.001. 12. 47.802. 13. 8.41. 14. .096.
15. 135.001. 16. 999.9991. 17. .999999. 18. 99.99999. 19. $462.334.
20. $53.01. 21. .00063. 22. 103.819516. 23. 4.726 cubic inches.
24. 46.274 cubic inches

Exercise XLIV.—(Page 51).—**1.** .56. **2.** .1344. **3.** .0576. **4.** .0018. **5.** .172. **6.** 24.5828. **7.** .23328. **8.** .0000072. **9.** 2.7. **10.** 84.240. **11.** 4.55625. **12.** 19135.97504. **13.** .8512. **14.** 37.496. **15.** .00119025. **16.** .68. **17.** 5. **18.** .00076. **19.** 73.3212. **20.** 8366.0166. **21.** 271.91443. **22.** .288008. **23.** 5.251. **24.** 732.09636. **25.** 37.13472. **26.** 261.319296.

Exercise XLV.—(Page 52).—**1.** $27.375. **2.** 13269.9 tons. **3.** 3336.6075 inches. **4.** 9.676128 yards. **5.** .3125. **6.** 220.43504386434. **7.** 8.4375 lbs. **8.** 268.269888 grains. **9.** 2218.192 cubic inches. **10.** $31.25.

Examination Paper, No. 8.—(Page 53).—**1.** 84.5664. **2.** $1.50. **3.** 63. **4.** $370.44. **5.** .02156580. **6.** $\frac{1}{3}$. **7.** $1\frac{1}{4}$. **8.** $60.40. **9.** $1.68. **10.** $550.

Exercise XLVI.—(Page 55).—**1.** 3.2. **2.** 4.8. **3.** .216. **4.** 3.07. **5.** .5399. **6.** .0278. **7.** .00005. **8.** .00024. **9.** .00004. **10.** .06. **11.** .00008. **12.** .000234. **13.** .000063. **14.** .00001. **15.** .0359.

Exercise XLVII.—(Page 56).—**1.** 3.07. **2.** 50.6. **3.** 1600. **4.** 6.446875. **5.** 12.4. **6.** .00075. **7.** .016125. **8.** .20024. **9.** 10000. **10.** .127. **11.** 850. **12.** .0065839. **13.** 2.0162. **14.** 80000. **15.** 1.173. **16.** .35969. **17.** .0029. **18.** 129. **19.** 3.59. **20.** 1527420.

Exercise XLVIII.—(Page 56).—**1.** 3.472222+. **2.** .012332+. **3.** 1283.655846+. **4.** 9.974564+. **5.** .000092+. **6.** 166.66666+. **7.** .069525+. **8.** 401.771749+. **9.** 37.137317+. **10.** .149274+. **11.** 7125890.736342+. **12.** .121764+.

Exercise XLIX.—(Page 57).—**1.** .25. **2.** .5. **3.** .6. **4.** .875. **5.** .9. **6.** 1.5. **7.** .04. **8.** .53125. **9.** .012. **10.** .0031.5. **11.** .02125. **12.** .59375. **13.** .14375. **14.** 64.625. **15** .088 **16.** 17.0875. **17.** 17.125. **18.** .17125. **19.** 5.781875. **20.** .3088. **21.** 4.0875. **22.** .00625. **23.** .00544. **24.** .088.

Examination Paper No. 9.—(Page 57).—**1.** 604001049.000000000063. **2.** A $6; B $9. **3.** 8 barrels. **4.** $\frac{1}{3}$. **5.** 108.8. **6.** A's $24; B's $36. **7.** A $60; B $120; C $240. **8.** 96. **9.** 32.54 ounces. **10.** $3125.

Examination Paper, No. 10.—(Page 58).—**1.** 3.6240002416. **2.** 172.8. **3.** $\frac{2475}{10000}$. **4.** .37 cubic inches. **5.** 1008. **6.** 288 bushels. **7.** $63000. **8.** 56. **9.** $\frac{11}{15}$. **10.** $\frac{1}{1250}$.

Exercise L.—(Page 58).—**1.** 6744.2009. **2.** .249975. **3.** 5.8493+. **4.** 1.0015+. **5.** .3144. **6.** .008. **7.** $\frac{33}{4000}$. **8.** 186.66 miles. **9.** 21120 times. **10.** $\frac{1}{10}$ of a bushel. **11.** 218.56957680. **12.** 27 suits. **13.** $320. **14.** .0198. **15.** $0.108. **16.** $243. **17.** $26\frac{2}{3}$. **18.** $16. **19.** $0.055+. **20.** 800 bushels. **21.** $3\frac{1}{3}$. **22.** $20\frac{5}{8}$. **23.** $300. **24.** B is $1\frac{1}{4}$ of A's; A's is $\frac{4}{5}$ of B's.

Exercise LI.—(Page 62).—1. 2036.529^{m}. 2. 71991.032^{m}. 3. 40789.94^{m}. 4. 822.968^{m}. 5. $243.38. 6. $143.685. 7. 2999.4^{m}. 8. 113846.4^{m}. 9. 7780.55^{m}. 10. 2683.746^{m}. 11. 1553.42^{m}. 12. 9263.4^{m}. 13. 222428^{m}. 14. 4.6844^{m}. 15. 196.6^{m}. 16. 27.35^{m}. 17. 7.83^{m}. 18. 2.965^{m}. 19. 318.9^{m}. 20. 456.79^{m}. 21. $7.8125. 22. $19.43+. 23. $13.48+. 24. 3000 steps. 25. 21.68^{m}. 26. 5 suits and 2.75^{m} over. 27. 1200 times. 28. 706–1.

Exercise LII.—(Page 65).—1. 1200^{qdm}. 2. 22.66^{qm}. 3. $30. 4. $14. 5. 68^{qm}. 6. 27.495^{qm}. 7. 4.5^{m}. 8. 5.12^{m}. 9. 864000^{qcm}. 10. .287659^{qm}. 12. 1.82^{m}. 13. 200000^{ca}; 2000^{a}; 20^{Ha}. 14. $2263.20. 15. $350.

Exercise LIII.—(Page 68).—1. 80400000$^{cu\ cm}$. 2. 140300000000$^{cu\ mm}$. 3. .009837216$^{cu\ m}$. 4. 12000$^{cu\ cm}$. 5. 1.43582$^{cu\ m}$. 6. One million times. 7. 44$^{cu\ m}$; 44st. 8. 5.21$^{cu\ m}+$. 10. 191.88$^{cu\ m}$. 11. $81. 12. 65st. 13. 4^{m}. 14. 20.8^{m}. 15. $31.59.

Exercise LIV.—(Page 69).—1. 18400^{l}. 2. 7243^{cl}. 3. 896^{l}. 4. 3997.76^{Kl}. 5. $108. 6. 3^{Kl}. 7. 800 min. or 13 hr. 20 min. 8. 12 cents. 9. $4. 10. $3.66.

Exercise LV.—(Page 70).—1. 17400^{g}. 2. 6830^{cg}. 3. 63^{T}. 4. $26.40. 5. 168750^{g}. 6. 784.6^{Kg}. 7. 5740.925^{g}. 8. 308^{Kg}. 9. 54.432^{T}. 10. 14 cents.

Examination Paper, No. 11.—(Page 71).—1. 4784.515^{m}. 2. 0. 3. $1.13. 4. 1.44^{Km}. 5. 34.15^{m}. 6. $1.911. 7. 49 cents. 8. Bring to common denominators and place difference of numerators over common denominator. 9. 7^{m}. 10. 16.63^{ar}.

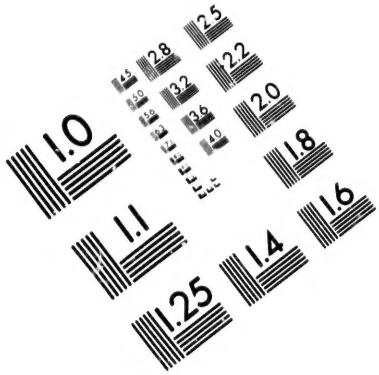
Examination Paper, No. 12.—(Page 71).—2. 1293^{g} or 1.293^{Kg}. 3. .562^{m}. 5. 20^{Kg} of water. 6 $1084.875. 7. $84. 8. $208. 9. $7.3125. 10. 88.9+^{Kg}.

Examination Paper, No. 13.—(Page 72).—1. 12 men. 2. 3 hr. 40 min. 4. 56700^{l}. 5. 24 cents. 6. Divide their sum by the number of addends. 7. 960^{m}. 8. 120^{a}. 9. 124.8^{Kg}. 10. $2284.75.

Exercise LVI.—(Page 77).—1. 52 T. 14 cwt. 52 lb. 7 oz. 2. £79 16s. 10½d. 3. 42 mi. 2 fur. 14 rd. 1 ft. 11 in. 4. 111 bu. 1 pk. 1 gal. 1 pt. 5. 261 da. 9 hr. 4 min. 15 sec. 6. 1170 A. 82 sq. rd. 29 sq. yd. 127 sq. in. 7. 330 A. 86 sq. rd. 11 sq. yd. 2 sq. ft. 15 sq. in. 8. 515.45^{m}. 9. 36 T. 14 cwt. 14 lb. 3 oz. 9 dr. 10. 948 cu. yd. 2 cu. ft. 400 cu. in.

Exercise LVII.—(Page 78).—1. £230 12s. 6d. 2. £303 12s. 6d. 3. 665.604672^{Kg}. 4. 1276 A. 136 sq. rd. 21 sq. yd. 7 sq. ft. 72 sq. in. 5. 66 mi. 2 fur. 10 rd. 6. 2723 lb. 14 oz. 7. 2 mi. 1 fur. 12 rd. 2 yd.; 29 gal. 3 qt.

Exercise LVIII.—(Page 79).—1. $75.75. 2. £18 17s. 11½d. 3. 38.07^{m}. 4. 38 yd. 9 in. 5. 16 cwt. 52 lb. 7 oz. 6. 13 A. 44 sq. rd. 10 sq. yd. 5 sq. ft. 36 sq. in. 7. 1100.73^{m}. 8. 5 bu. 2 pk. 1 qt. 1 pt. 9. 1 ft. 5 in.



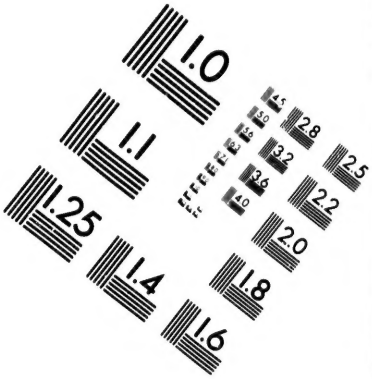
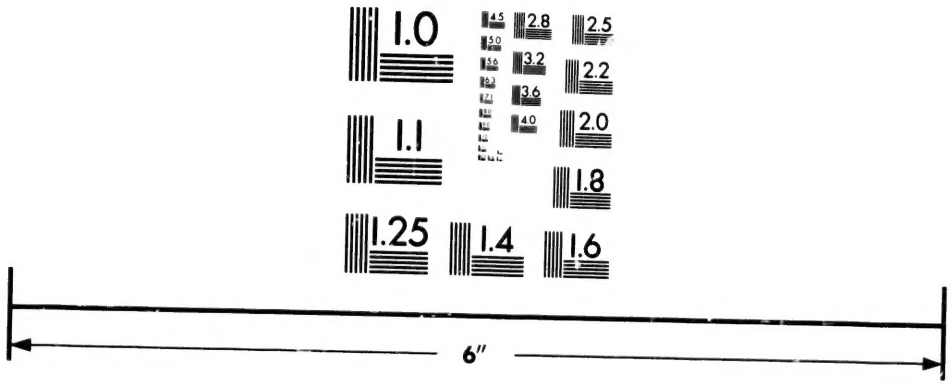
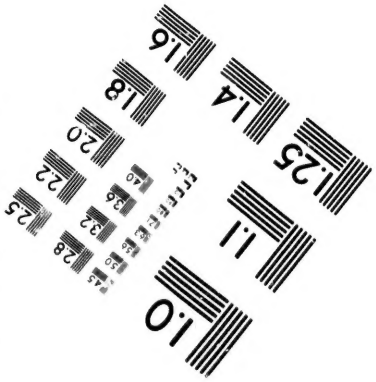
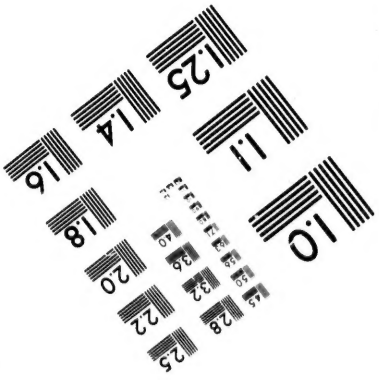


IMAGE EVALUATION TEST TARGET (MT-3)







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Exercise LIX.—(Page 81).—**1.** 3 hr. 37 min. 5 sec.
2. 2 mi. 2 fur. 19 rd. $8\frac{4}{5}$ in. **3.** 13 bu. 3 pk. 1 gal. $2\frac{1}{15}$ qt.
4. 3 mi. 2 fur. 26 rd. 3 yd. 2 ft. **5.** 1 cwt. $62\frac{4}{7}$ lb. **6.** 6.420^{m}.
7. 3 mi. 30 rd. 4 yd. 2 ft. $1\frac{4}{7}$ in. **8.** £2 17s. $9\frac{1}{2}$d.
9. 17 T. 14 cwt. 93 lb. 14 oz. **10.** 18 bu. 1 pk. 1 gal. $\frac{1}{14}$ qt.

Exercise LX.—(Page 82).—**1.** 3337 far. **2.** 297599 far.
3. £2 1s. 8d. **4.** £800 $1\frac{1}{4}$d. **5.** £41 13s. 4d. **6.** 231336d.
7. 24288 oz. **8.** 1543424. **9.** 7525612 dr. **10.** 28000 oz.
11. 13908 min. **12.** 1790 pt. **13.** 115 bu. 1 qt. 1 pt. **14.** 3445200 sec.
15. 31556926 **16.** 3 T. 2 cwt. 52 lb. 12 oz. **17.** 370800 sec.
18. 61700^{cm}. **19.** 4 T. 7 lb. 12 oz. 10 dr.

Exercise LXI.—(Page 84).—**1.** 37 mi. 6 fur. 35 rd. 2 yd. 1 ft. 11 in.
2. 1306872 in. **3.** 788200^{mm}. **4.** 96.012^{Kg}. **5.** 3 lb.
6. 4 lb. 10 oz. 6 dwt. 16 gr. troy, 4 lb. 10 oz. 2 dr. 2 scr. ap., 4 lb. av.
7. 1167300 sec. **8.** 20067725 sq in. **9.** 2000^{g}, 30864 gr.
10. 1 A. 95 sq. rd. 2 sq. yd. 2 sq. ft. 100 sq. in.
11. 7206 bu. 1 pk. 1 gal. **12.** 56209280. **13.** 34015.68 in. **14.** 2100^{m}.
15. 1230 in., 31.24^{m}. **16.** $16\frac{3}{5}\frac{0}{2}\frac{3}{8}\frac{1}{0}$ miles. **17.** 18 lots and 40 sq. yd. over.
18. $6.42. **19.** $40.97+. **20.** 46464 times.